STEP - Integration Exercises (1 page; 28/9/18)

(1) If $\int_{-a}^{a} f(x)\,dx = b$, find $\int_{-a}^{a} f(-x)\,dx$

(2) Explain the following 'paradox':

$\int \frac{1}{2x}dx = \frac{1}{2}\int \frac{1}{x}dx = \frac{1}{2}lnx + C$

but $\int \frac{1}{2x}dx = \frac{1}{2}ln(2x) + C$ (by the reverse Chain rule)

(3) Given that $\int \frac{1}{x}dx = \ln x$ for $x > 0$, show that $\int \frac{1}{x}dx = ln|x|$ for all $x \neq 0$

(4) Given that $f(x)$ has a maximum on the interval $0 \leq x \leq \frac{1}{2}$ at $x = x_0$, show that $\int_0^x f(t)dt \leq \frac{1}{2}f(x_0)$ whenever $0 \leq x \leq \frac{1}{2}$