STEP Exercises - Integers (2 pages; 12/9/18)

(1) Can n^3 equal $n + 12345670$ (where n is a positive integer)?

(2) Find all positive integer solutions of the equation

$xy - 8x + 6y = 90$

(3) Show that $f(h) = h\left(\frac{N}{N+1}\right)^h$ is maximised when $h = N$ (h is an integer).

(4) **D.** The smallest positive integer n such that

$$1 - 2 + 3 - 4 + 5 - 6 + \cdots + (-1)^{n+1}\, n \geqslant 100,$$

is

(a) 99, (b) 101, (c) 199, (d) 300.

(5) **J.** The number of *pairs* of *positive integers* x, y which solve the equation

$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 2^{30}$$

is

(a) 0, (b) 2^6, (c) $2^9 - 1$, (d) $2^{10} + 2$.

(6) **J.** Let a, b, c be positive numbers. There are *finitely* many *positive whole* numbers x, y which satisfy the inequality

$$a^x > c\,b^y$$

if

(a) $a > 1$ or $b < 1$.
(b) $a < 1$ or $b < 1$.
(c) $a < 1$ and $b < 1$.
(d) $a < 1$ and $b > 1$.

(7) **I.** The function $S(n)$ is defined for positive integers n by

$$S(n) = \text{sum of the digits of } n.$$

For example, $S(723) = 7 + 2 + 3 = 12$. The sum

$$S(1) + S(2) + S(3) + \cdots + S(99)$$

equals

(a) 746, (b) 862, (c) 900, (d) 924.