STEP: Further Exam Technique (7 pages; 11/8/16)

(1) Using information given in the question

Examples

(a) "Given that $c \neq 0$"

(b) "... assuming no friction" (for a car on a banked track)

(c) "... a fixed frictionless pulley" or "... in the direction of greatest slope on the plane"

(d) "Given that the magnitude of the impulsive force on the lift due to tension in the cable is equal to the magnitude of the impulsive force on the counterweight due to tension in the cable ..." [STEP 3, 2006, Q11]

There are several possibilities:

(i) The information is just to satisfy pedants, who might claim that the problem was insoluble without the given information. The examples in (c) come under this heading.

(ii) The information is there to remove a complication. Case (a) is in this category. However, we should make reference to the fact that $c \neq 0$ when, for example, dividing by c.

(iii) In (b), the absence of friction is a special case (chosen to simplify the problem).

(iv) In (d), the information is there as a hint. It can in fact be inferred from the situation in question (though there may not be time in the exam to examine it in depth).

(2) Algebra

(i) Algebra can feature in many questions (including Mechanics and Probability). Examiners are always bemoaning candidates' shortcomings regarding algebra. The amount of algebraic working required can be quite extensive (in general, much greater than for the MAT paper). At A Level, if the algebra becomes complicated it is usually the case that you have gone wrong somewhere, but this isn't true for STEP.

(ii) Compare the number of unknowns with the number of equations.

It may be the case that only ratios are needed (see 2006, P2, Q10).

If the the number of unknowns is greater than the number of equations then something fortuitous must occur (unless you have made a mistake).

If a polynomial equation is to hold for all values of x, then of course we can equate coefficients of powers of x (ie we really have several equations).

(iii) Rather than attempting to solve a large number of simultaneous equations, look for a way of eliminating variables as you go along.

(iv) Use letters to represent recurring expressions.

(3) Complications

A recurrent theme is the presence of complications in the questions.

Examples

(a) recognising the range of values for which a result is valid (eg if a Binomial expansion is involved)

(b) use of ln|x| (rather than just lnx)

(c) $\sqrt{x^2} = |x|$ (not x) [Note that, by convention, $\sqrt{4}$ means 2; not ± 2]

(d) avoiding division by zero

A typical pattern is that the 1st part of a question is straightforward, but a modification introduced into the 2nd part means that a complication has to be taken into account (eg the case where $x = 0$ may have to be treated separately, to avoid a division by zero). The consequence of missing this complication may just mean that full marks are not obtained, but it could result in the answer being worthless.

Even if a complication cannot be acted on fully, it is probably worth at least showing an awareness of the problem. Borderline candidates (those who have just missed out on the required STEP grades) will have their scripts examined by the tutors of the college (or university) that made them the offer, and they will be looking for things that set them apart from other candidates with similar scores. Having said this, however, it is stated by the examiners that marks are only awarded for doing work, not for saying what you **would** do.

A word of warning though: Sometimes you may identify an issue which you feel needs addressing, but it turns out that the examiners are not interested for some reason (eg if the question is sufficiently demanding in other respects). You may want to save discussion of this issue until the end of the exam (if there is nothing better to do then).

(4) Showing where statements come from

Example: Suppose a candidate has written the following:

$xy < xz$

$y < z$

$x > 0$

As it stands, it is not clear whether $y < z$ is supposed to lead on from

$xy < xz$ (which would be incorrect, as x could be negative), or whether $y < z$ is a result established earlier (or perhaps stated in the question). Likewise, is $x > 0$ being deduced, or brought in from somewhere else?

To avoid any uncertainty, each statement really needs something to show where it comes from.

A revised version of the above might be:

From (1), $xy < xz$

Also, the question states that $y < z$

Hence $x > 0$

(5) Spotting shortcuts

These are often possible by the use of a symmetry argument, or a result established earlier in the question.

Be wary of any work that just repeats a method already applied; ie look for a shortcut (examiners will be reluctant to allocate many marks for repeating an idea). See STEP 2, 2008, Q10.

Also, when you have discovered a suitable method, look for an improved version.

(6) Common Pitfalls

(i) not considering all cases (especially to avoid division by zero)

(ii) not justifying an argument fully (eg " $a^3 < b^3 \Leftrightarrow a < b$ " probably needs "because $y = x^3$ is an increasing function")

(iii) not using $\Leftrightarrow$

For “if and only if” proofs, it may be sufficient to indicate that the line of reasoning is reversible, by using the $\Leftrightarrow$ symbol (assuming that this is the case).

(7) Checking

This needs to be an ingrained habit (ie not just saved for the exam).

For example, SPARE Q (think of Scrabble, where you generally have a Q left at the end):

S: substitution (does the answer satisfy the original equation(s)?)

P: proofreading (looking over each line after it has been written – to eliminate elementary slips; eg involving minus signs)

A: alternative method (eg for checking numerical work; 0.04x0.06 could be worked out by converting the numbers into fractions, as well as by counting decimal places)

R: reasonableness (is there anything suspicious about the answer?)

E: estimate (eg rough check on 243 x 47: 240 x 50)

Q: read the question again (eg before embarking on the solution, and at the end - in case any additional task has been overlooked)
[Note: zero marks are usually awarded for not using a specified method]

It is also a good idea to re-read the question if you find yourself getting bogged down in awkward algebra, or if you don't seem to be getting anywhere.

(8) Miscellaneous

(i) Explanations

It can be a good idea to describe what you are doing, for the examiner's benefit. (However, the examiners have said that credit can't be given for a description of what you would do, if you had more time.)

(ii) Save a straightforward task until the end of the exam: it is probably more efficient to be doing a simple task in the last 10 minutes, rather than frantically looking through the paper for something sensible to do or check.

(iii) The first part of a STEP question may appear to be much too easy, and a trap may be suspected. Sometimes there *is* a complication to be taken into account, but often it is just intended as a gentle introduction, to point you in the right direction. It may, for example, have been added in as an afterthought – in order not to make the question too difficult. The examiners are generally keen for students to be able to at least start a question.

The last part of a STEP question isn't necessarily any harder than the earlier parts - especially once you have got on the question-setter's wavelength. Also, the last part might simply be the final (easy) stage in establishing an interesting result.

(iv) STEP 2 is intended to be harder than STEP 1. Both only involve minimal material from the Further Maths syllabuses (including Proof by Induction and Inequalities). Because STEP 3 opens up the possibility of questions on any Further Maths topic (in the STEP syllabus) it may require more preparation of topics, but it can be argued that STEP 3 questions are often easier than STEP 2 – once the topic in question has been studied - on the basis that the STEP 2 paper has a smaller fund of suitable topics, and therefore tends to feature harder questions on familiar themes.

(v) An observation made by the examiners themselves is that an unusual question, or one that quotes an obscure-looking result, often turns out to be simpler than a more standard question. Invariably no prior knowledge of the quoted result is in fact needed. The examiners are effectively rewarding candidates for coping with a new idea.

(vi) Note the following wording included with the syllabus:

"Normally, a candidate who answers at least four questions well will be awarded a grade 1. The marking scheme for each question will be designed to reward candidates who make good progress towards a complete solution."

It isn't clear what is meant by this. It could just mean that there is a strong correlation between answering four questions well and being awarded a grade 1; or it could - perhaps - mean that some discretion is available to the examiners when awarding a grade 1.

Candidates' scripts are available to college and university tutors, for them to judge the quality of solutions in borderline cases.