STEP Exercises - Turning points, Points of inflexion & Cubics (2 pages; 17/9/18)

(1) What can be said about the graph of $f(x)$ if $(x-a)^n$ is a factor of $f(x)$, where $f(x)$ is a polynomial function and $n \in \mathbb{Z}^+$?

(2) Find the turning points of $y = (x^2 - 4x + 3)^2$

(3) (i) For $f(x) = ax^3 + bx^2 + cx + d$, what is the x-coordinate of the point of inflexion?

(ii) Give examples of cubic functions for which the PoI is at the Origin, and the gradient at the Origin is (a) 1 (b) -1. How do the shapes of the two graphs differ?

(4) Cubics

(i) What possible shapes might a cubic have (ignoring its position relative to the axes)?

(ii) How many stationary points does the cubic function,

$f(x) = x^3 + x^2 - 2x + 3$ have?

(iii) What is the condition for there to be 2 stationary points for the general cubic $f(x) = ax^3 + bx^2 + cx + d$?

(iv) For $f(x) = ax^3 + bx^2 + cx + d$, find the x-coordinate of any turning points of the gradient.

If the cubic has turning points, how could they be used to find the point of inflexion?

(v) For $f(x) = ax^3 + bx^2 + cx + d$, find conditions for the shape of the curve to be each of the 3 possibilities shown in (i), by considering the gradient at the point of inflexion.