STEP Exercises - Logs (sol'ns) (3 pages; 22/9/18)

(1) Show that $\log(4-\sqrt{15}) = -\log(4+\sqrt{15})$

Solution

$\log(4-\sqrt{15}) = -log\left(\frac{1}{4-\sqrt{15}}\right) = -log\left(\frac{4+\sqrt{15}}{16-15}\right) = -\log(4+\sqrt{15})$

[or $\log(4-\sqrt{15}) + \log(4+\sqrt{15}) = \log\{(4-\sqrt{15})(4+\sqrt{15})\}$

$= \log(16-15) = 0$]

(2) If $k = log_{24}12$, write the following in terms of k:

(a) $log_{24}2$ (b) $log_{24}6$

Solution

(a) $log_{24}2 = log_{24}\left(\frac{24}{12}\right) = log_{24}24 - log_{24}12 = 1-k$

(b) $log_{24}6 = log_{24}\left(\frac{12}{2}\right) = log_{24}12 - log_{24}2 = k-(1-k) =$
$2k-1$

[or $log_{24}6 = log_{24}\left(\frac{24}{4}\right) = log_{24}24 - log_{24}4 = 1 - log_{24}(2^2)$

$= 1 - 2log_{24}2 = 1 - 2(1-k) = 2k-1$]

(3) Write $log_2 3$ in terms of logs to the base 10

Solution

Method 1

Standard result: $log_a b\ log_b c = log_a c$

[a is raised to the power of $log_a c$ in order to get to c;
alternatively, raise a to the power of $p = log_a b$, to get to b, and

then raise b to the power of $q = log_b c$, to get to c; thus $a^p = b$ and $b^q = c$, which gives $(a^p)^q = c$, and hence $a^{pq} = c$, so that $log_a c = pq = log_a b\ log_b c$]

Then $log_b c = \frac{log_{10} c}{log_{10} b}$, so that $log_2 3 = \frac{log_{10} 3}{log_{10} 2}$

Method 2

Set up an equation, as follows:

Let $log_2 3 = x$

[The advantage of creating an equation is that we then have something that can be manipulated.]

$\Rightarrow 3 = 2^x$

$\Rightarrow log_{10} 3 = x log_{10} 2$

$\Rightarrow log_2 3 = x = \frac{log_{10} 3}{log_{10} 2}$

(4) Is $log_2 3 > \frac{3}{2}$?

Solution

$log_2 3 > \frac{3}{2} \Leftrightarrow 3 > 2^{\frac{3}{2}}$ (as $y = 2^x$ is an increasing function)

$\Leftrightarrow 3^2 > 2^3$

So answer is Yes.

(5) Simplify $\frac{log_x b}{log_x a}$

Solution

Without loss of generality , we can suppose that $x < a < b$, with $x^p = a$

and $a^q = b$, so that $x^{pq} = b$

Then $\frac{log_x b}{log_x a} = \frac{pq}{p} = log_a b$

[or $log_x a . log_a b = log_x b$: in terms of powers, p takes you from x to a, and q takes you from a to b; so pq takes you from x to b]