STEP Exercises - Inequalities (2 pages; 6/10/19)

(1) Are the following true or false?

(i) $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

(ii) $a < b \Rightarrow a^2 < b^2$

(iii) $a < b \ \& \ c < d \Rightarrow a + c < b + d$

(iv) $a < b \ \& \ c < d \Rightarrow a - c < b - d$

(2) Prove or provide a counter-example for the conjecture

$x > a \ \& \ y > b \Rightarrow xy > ab \quad (a, b \ real)$ in each of the following cases:

(i) $a > 0, b > 0$ (ii) $a < 0, b < 0$ (iii) $a > 0, b < 0$

(3) Prove that $a + b < 1 + ab$ if $a > 1 \ and \ b > 1$

(4) Prove that $\frac{a}{b} < \frac{a+c}{b+c}$ where $a, b, c > 0 \Leftrightarrow a < b$

(5) Let $x, y \ \& \ z$ be positive real numbers.

(i) If $x + y \geq 2$, is it necessarily true that $\frac{1}{x} + \frac{1}{y} \leq 2$?

(ii) If $x + y \leq 2$, is it necessarily true that $\frac{1}{x} + \frac{1}{y} \geq 2$?

(6) Assuming that $sin^2\theta + cos^2\theta = 1$, but without using any compound angle results, show that $sin\theta cos\theta \leq \frac{1}{2}$

(7) Which is larger: $\frac{\sqrt{7}}{2}$ or $\frac{1+\sqrt{6}}{3}$ (without using a calculator)?

(8) Is $\frac{6}{7} < \frac{2}{\sqrt{5}}$?

(9) Show that $e^3 > 4e^{\frac{3}{2}}$

(10) Is $log_2 3 > \frac{3}{2}$?

(11) Use differentiation to show that $lnx \geq 1 - \frac{1}{x}$ for $x > 0$