

STEP - Differentiation Exercises (sol'ns) (2 pages; 28/9/18)

(1) If $f(x) = x^2$, what is $f'(3x)$?

Solution

Method 1

Note that the differentiation is wrt $3x$ (rather than x).

Let $u = 3x$. Then $f'(3x) = f'(u) = \frac{d}{du}(u^2) = 2u = 2(3x) = 6x$

Method 2

$f'(x) = 2x \Rightarrow f'(3x) = 2(3x) = 6x$

(2) Find $\frac{d}{dx}(a^x)$

Solution

Method 1

Let $a = e^b$. Then $\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{bx}) = be^{bx} = \ln a \cdot a^x$

Method 2

Let $y = a^x$. Then $\ln y = x \ln a$,

and, differentiating wrt x gives $\frac{1}{y} \frac{dy}{dx} = \ln a$, so that $\frac{dy}{dx} = \ln a \cdot a^x$

(3) Show that $\frac{d}{dx}(x^x) = x^x(1 + \ln x)$

Solution

Let $y = x^x$

Then $\ln y = x \ln x$

$$\text{and } \frac{1}{y} \frac{dy}{dx} = \ln x + x \left(\frac{1}{x} \right)$$

$$\text{so that } \frac{dy}{dx} = y(\ln x + 1) = x^x (1 + \ln x)$$

$$(4) \text{ Find } \frac{d}{dx} (x^{\sin x})$$

Solution

$$\begin{aligned} \frac{d}{dx} (x^{\sin x}) &= \frac{d}{dx} (e^{\ln x \cdot \sin x}) = e^{\ln x \cdot \sin x} \left(\frac{1}{x} \sin x + \ln x \cdot \cos x \right) \\ &= x^{\sin x} \left(\frac{1}{x} \sin x + \ln x \cdot \cos x \right) \end{aligned}$$

$$(5) \text{ Find } \frac{d}{dx} \log_a x$$

Solution

$$\frac{d}{dx} \log_a x = \frac{d}{dx} (\log_a e \cdot \ln x) = \frac{1}{x \ln a}$$