## STEP/Curve Sketching Q2 (14/6/23)

Sketch (i) $y=\sqrt{\sin x}$ and (ii) $y=(\sin x)^{\frac{1}{n}}$ for large positive integer $n$ (for $0 \leq x \leq \pi$ in both cases).

## Solution


(i) Note that, for $0<y<1, \sqrt{y}>y$

So, for $y=\sqrt{\sin x}$, the graph will hug the $y$-axis more than for $y=\sin x$.

Also, if $f(x)=\sqrt{\sin x}, f^{\prime}(x)=\frac{1}{2}(\sin x)^{-\frac{1}{2}} \cos x$, so that $f^{\prime}(0)=\infty$ (strictly speaking, it is 'undefined');
ie the graph is vertical at $x=0$ (and also $x=\pi$, by symmetry).
(ii) The effect is greater for larger $n$, and the graph tends to a rectangular shape.

