STEP 2014, P1, Q11 - Solution (4 pages; 15/11/19)

(i) Applying N2L to the particle of mass M,

$Mg - T = Ma_1$ (1) , where T is the tension in the string.

Then applying N2L to the particle of mass m:

$T - mg = ma_1$ (2)

Adding (1) & (2) gives $(M - m)g = a_1(M + m)$

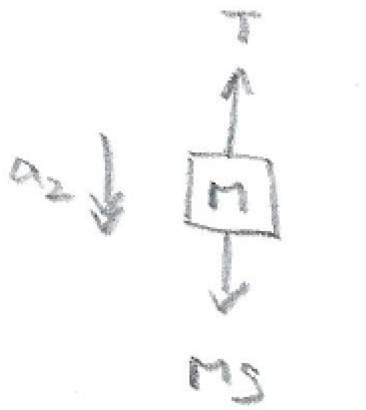
$\Rightarrow a_1 = \frac{M-m}{M+m}\, g$, as required.

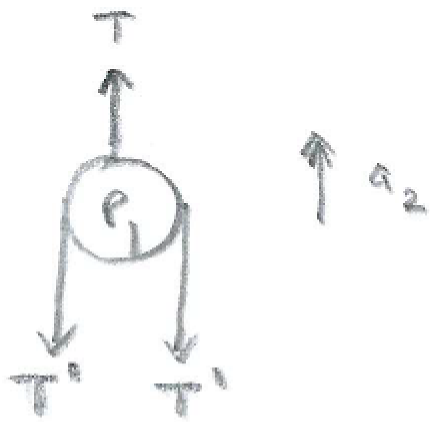
Then the force on the pulley $= 2T$,

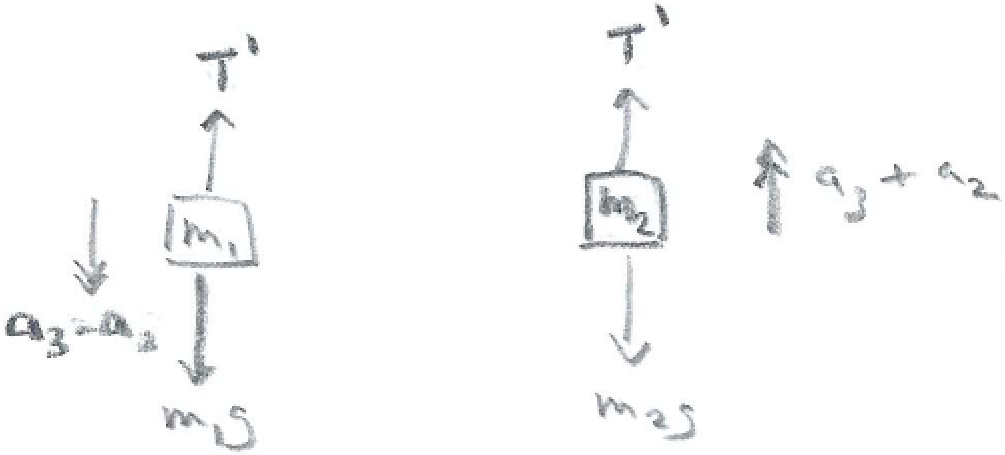
and from (2), $2T = 2m(a_1 + g) = 2mg\left(\frac{M-m}{M+m} + 1\right)$

$= 2mg\ \frac{M-m+(M+m)}{M+m} = \frac{4mMg}{M+m}$

(ii)







where a_3 is the acceleration of m_1 & m_2 relative to P_1

Applying N2L to the particle of mass M,

$Mg - T = Ma_2$ (3)

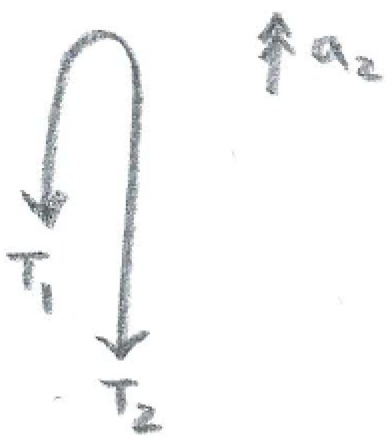
Applying N2L to P_1,

$T - 2T' = (0)a_2$ (as the pulley is assumed to be light; ie of negligible mass), so that $T' = \frac{1}{2}T$ (4)

[It can be taken for granted that the tension T' throughout the string connecting masses m_1 and m_2 is the same.

The argument would be as follows:

Applying N2L to the string, $T_1 - T_2 = (0)a_2$, where T_1& T_2 are the tensions at the two ends of the string, and the string is assumed to be of negligible mass (see diagram below).



Thus $T_1 = T_2$.]

Applying N2L to the particle of mass m_1,

$m_1 g - \frac{T}{2} = m_1(a_3 - a_2)$ (5)

and applying N2L to the particle of mass m_2,

$\frac{T}{2} - m_2 g = m_2(a_3 + a_2)$ (6)

Using (3) to eliminate T,

$(5) \Rightarrow m_1 g - \frac{M}{2}(g - a_2) = m_1(a_3 - a_2)$ (5')

and $\frac{M}{2}(g - a_2) - m_2 g = m_2(a_3 + a_2)$ (6')

To eliminate a_3, $m_2(5') - m_1(6')$ gives

$2m_1 m_2 g - \frac{M}{2}(g - a_2)(m_2 + m_1) = -2m_1 m_2 a_2$

Multiplying by 2,

$a_2\{M(m_2 + m_1 + 4m_1 m_2\} = Mg(m_2 + m_1) - 4m_1 m_2 g$

so that $a_2 = \frac{(M-4\mu)}{(M+4\mu)} g$, as required.

For the last part, $a_1 = a_2 \Leftrightarrow \frac{M-m}{M+m} g = \frac{(M-4\mu)}{(M+4\mu)} g$

$\Leftrightarrow \frac{M-m}{M+m} = \frac{M - \frac{4m_1 m_2}{m}}{M + \frac{4m_1 m_2}{m}}$

$\Leftrightarrow M^2 + \frac{4m_1 m_2}{m} M - mM - 4m_1 m_2$

$= M^2 - \frac{4m_1m_2}{m}M + mM - 4m_1m_2$

$\Leftrightarrow \frac{4m_1m_2}{m}M - mM = -\frac{4m_1m_2}{m}M + mM$

multiplying by $\frac{m}{M}$:

$\Leftrightarrow 4m_1m_2 - m^2 = -4m_1m_2 + m^2$

$\Leftrightarrow 8m_1m_2 = 2m^2$

$\Leftrightarrow (m_1 + m_2)^2 - 4m_1m_2 = 0$

$\Leftrightarrow (m_1 - m_2)^2 = 0$

$\Leftrightarrow m_1 = m_2$

Thus $a_1 = a_2$ if and only if $m_1 = m_2$.