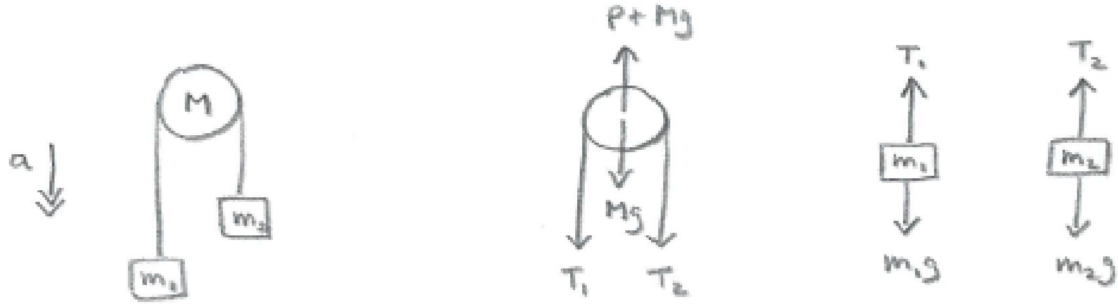
STEP 2012, Paper 3, Q9 – Solution (2 pages; 16/7/18)

(i)



N2L for the two particles gives:

$m_1g - T_1 = m_1a$ (1) & $T_2 - m_2g = m_2a$ (2)

(with obvious notation [which unfortunately would have to be defined in the exam])

[As the contact between the string and the pulley is rough, there is a frictional force (F) on the string. Considering the forces on the string, $T_1 - T_2 - F = ma$, where m, the mass of the string, is negligible, so that $T_1 = T_2 + F$; ie $T_1 \neq T_2$. In A Level questions, without this friction, the assumption is made that $T_1 = T_2$.]

For the pulley, total moments of external forces about O $= I\ddot{\theta}$,

so that $T_1r - T_2r = I\left(\frac{a}{r}\right) \Rightarrow T_1 - T_2 = I\left(\frac{a}{r^2}\right)$ (3)

[$\dot{\theta} = \frac{v}{r}$, so that $\ddot{\theta} = \frac{a}{r}$]

Also, resolving vertically for the pulley:

$P + Mg = T_1 + T_2 + Mg \Rightarrow P = T_1 + T_2$ (4)

(3) $\Rightarrow I = \frac{r^2}{a}(T_1 - T_2)$ (5)

$(1) \Rightarrow T_1 = m_1(g-a)$ (6)

$(2) \Rightarrow T_2 = m_2(g+a)$ (7)

$(4),(6),(7) \Rightarrow P = (m_1+m_2)g + a(m_2-m_1)$

$\Rightarrow a = \frac{P-(m_1+m_2)g}{m_2-m_1}$ (8)

$(5),(6),(7) \Rightarrow I = \frac{r^2}{a}\left(g(m_1-m_2) - a(m_1+m_2)\right)$

Then, from (8):

$$I = -\frac{r^2 g(m_1-m_2)^2}{P-(m_1+m_2)g} - r^2(m_1+m_2)$$

$$= \frac{r^2\{g(m_1-m_2)^2-(m_1+m_2)[(m_1+m_2)g-P]\}}{(m_1+m_2)g-P}$$

$$= \frac{r^2\{(m_1+m_2)P+g[(m_1-m_2)^2-(m_1+m_2)^2]\}}{(m_1+m_2)g-P}$$

$= \frac{r^2\{(m_1+m_2)P-4m_1m_2g\}}{(m_1+m_2)g-P}$, as required

(ii) Let I_0 & I_1 be the old and new moments of inertia.

The only change is that equation (3) becomes

$(T_1-T_2)r - C = I_1\left(\frac{a}{r}\right)$, as friction opposes motion (9)

Then, since $(T_1-T_2)r = I_0\left(\frac{a}{r}\right)$,

$$I_1 = \left\{I_0\left(\frac{a}{r}\right) - C\right\}\left(\frac{r}{a}\right) = I_0 - \frac{Cr}{a}$$

ie the new moment of inertia is smaller than the old value.

From (9), as $I_1 > 0, C < (T_1-T_2)r = m_1(g-a)r - m_2(g+a)r$

$= (m_1-m_2)rg - ar(m_1-m_2) < (m_1-m_2)rg$, as required

(as $m_1 > m_2$ & $a > 0$)