STEP 2007, Paper 3 – Notes (4 pages; 31/5/18)

See separate documents for Sol'ns.

1	2	3	4	5	6	7	8
Sol'n	Sol'n	Sol'n	N	N	N	Sol'n	N

9	10	11		12	13	14
N	N			N	N	

Q4 Often the questions with new material (here, the radius of curvature) turn out to be more straightforward than questions on familiar topics (as is the case here).

Notice how *everything* in this question (apart from the sketch of the curve – which is fairly simple anyway) is of a 'show that' nature (ie self-checking).

Tip for showing that $\rho = a\cot t$: first of all, evaluate the numerator & denominator of ρ separately. It turns out that the numerator is $(a\cot t)^3$. Then, since we need to show that the denominator is $(a\cot t)^2$, we can 'force' it into this form by taking out the (not immediately obvious) factor of $(a\cot t)^2$ from both terms in the expression, and showing that the remainder becomes 1.

You could be forgiven for not spotting the "standard differential of $\ln\tan\frac{t}{2}$ " (cosec t), referred to in the official sol'n.

Q5 If you obtain the correct expression for $\frac{d^3y}{dx^3}$ ($\frac{4}{r^3}\cosh 3\theta$), the pattern

2, -2, 4 might look as if you have made a mistake (eg perhaps the 4 should be a 2?), and a generalised formula is not obvious (assuming the 4 is correct)

However, applying proof by induction on $\frac{coshn\theta}{r^n}$ (ie ignoring the multiple) makes the pattern clear.

For future reference, the pattern 2, 2, 4 ... is 2 (0!, 1!, 2!, ...) in disguise.

Q6 The algebra of conjugate numbers is a nice sub-topic of complex numbers. This question illustrates 3 useful devices:

(i) creating the equation $zz^* = r^2$

(ii) rotation through $\frac{\pi}{2}$ rad is represented by multiplying by ki, where k is real

(iii) the operation of taking the conjugate of both sides of an equation, using the various rules; eg (as in this question):

$p - q = ki(r - s) \Rightarrow p^* - q^* = k(-i)(r^* - s^*)$, where k is real again

Q8 In part (ii), you have to have faith that a similar substitution to that in (i) will in fact work. There is then the question "how similar"? In STEP, the simplest possible interpretation usually turns out to be the one that works.

Q9 For complicated Mechanics questions, the use of Conservation of Energy is strongly recommended (rather than forming equations from forces). In this question, the CoE approach is suggested anyway by giving the expression for the PE of the spring.

Q10 For the 1st part, there are two possible approaches: either produce the usual expressions for horizontal and vertical axes (say X & Y) and convert into the x-y form (this involves compound angles), or resolve gravity into x and y components (which is much simpler).

This is the usual STEP pattern: the 1st approach might only get you a 2(unless you were very quick), whereas the 2nd approach is probably needed for a 1 (when extended to the other questions). So, the moral is: having spotted something, look further! (Though of course there might not be anything to find – or you might not spot it. But it might prove fruitful for a couple of questions, and that could make the difference between completing enough questions and running out of time.)

For the particle to return along the same path, just consider how this could actually happen: ie it would have to be falling perpendicular to the x-axis. As usual, a very simple idea is involved.

'largest possible value' implies finding a stationary point (for $3\sin\varphi + \operatorname{cosec}\varphi$)

Q12 The key to this question is conditioning on the value of N

Thus $E(Y) = \sum_{r=1}^{2n-1} Prob(N = r).[E(Y)|N = r]$

Q13 The algebra in (iii) - solving 3 awkward simultaneous equations – is a reminder that there isn't always a simple way of answering a STEP question. The official solutions contain an obvious typo in (iii): the expected distance covered in 1 jump is p+2q (not q+2p)