STEP 2007, Paper 2 – Notes (4 pages; 23/5/18)

See separate documents for Sol'ns.

1	2	3	4	5	6	7	8
Sol'n	Sol'n	Sol'n	Sol'n	N	N	N	N

9	10	11		12	13	14
N	N	N		N	N	N

Q5 The presence of the (large) number 2007 is a hint that the function will be periodic. (With the benefit of hindsight,) this can only really mean that $f^3(\mathrm{x}) = \mathrm{x}$. (Obviously this doesn't constitute a rigorous proof!)

In (iii), the presence of the quantities $\frac{\sqrt{3}}{2}, \frac{1}{2}$ & $\sqrt{1-t^2}$ suggests that sines and cosines are involved.

In the official solution there is a discussion of an alternative answer –presumably overlooked when the question was set!

Q6 As there are no special methods in the Further Maths syllabus for dealing with differential equations involving $\left(\frac{dy}{dx}\right)^2$, the only option is to do something simple: making the substitution $\mathrm{u} = \frac{dy}{dx}$ in this case.

Substitutions are a common device for getting you out of trouble. Often it's a case of not knowing that the substitution will work, but that there is only one likely substitution – assuming that this is the correct approach.

Q7 This question shows that the examiners are trying to help you: the suitable function here is just the ln x considered earlier.

You are probably expected to demonstrate that the value obtained in (ii)(b) is in fact a minimum.

Q8 This question highlights several useful vector techniques:

(a) If P lies on AB, then $\overrightarrow{OP}$ can be denoted by λ**a** + (1-λ)**b** (if **a** = $\overrightarrow{OA}$ etc)

(b) If $\overrightarrow{OQ} = \lambda\mathbf{a} + \mu\mathbf{b}$ and R lies on OQ (or OQ extended), then $\overrightarrow{OR}$ can be denoted by k(λ**a** + μ**b**)

(c) If p**a** + q**b** = r**a** + s**b**, then p = r and q = s (provided O, A & B are not collinear – ie not on the same line); ie this is not just true for **a** = **i** and **b** = **j**

These enable equations to be created and then solved. In particular, the extra parameter introduced by (b) is outweighed by the ability to equate coefficients in (c).

Q9 In (ii), having considered $\frac{dw}{d\alpha} = 0$ and found the required value of cosα, there is no need to work out $\frac{d^2w}{d\alpha^2}$: you can just observe that w=0 when α=0 & π/2, and therefore w has a maximum between these values (ie rather than a minimum).

Q10 The 'standard approach' mentioned in the official solutions doesn't in fact seem to be applicable, but rotational equilibrium allows P to be established, from which the condition on μ follows, based on the fact that there is no slipping.

The 'refinement' in this question is that there are two configurations, so that P can be acting in either direction. Otherwise, the presence of the moduli signs is worrying, because of the lack of any obvious squares (which might lead to two answers). The second configuration can be discovered by finding approximately when $\frac{9}{8} - \frac{1}{2}\cot\alpha = 0$ (in case you didn't 'remember' that P could be either direction).

Some marks are obviously available for working out the centre of mass of the object. If you weren't able to see how to tackle the question, this bit

could have been saved for the last 5 minutes of the exam (if you had nothing else to do , and hadn't been able to tackle a more promising question).

Q11 This question seems to contain a couple of uncertainties/ambiguities.

First of all, the statement "in a direction that makes an angle 60^0 with OB". The question only seems to be do-able if 'direction' is taken to mean 'direction on the ground'. If instead we are considering the (3D) angle between the trajectory of the particle and OB, note that this depends on the angle of elevation (here arctan ½): to see this, consider what happens as the angle of elevation reaches 90^0 - the angle between the trajectory and OB is now 90^0 (whereas it was 60^0 when the particle was on the ground).

Oddly, the question qualifies the bearing from O as 'horizontal' - suggesting that you could have a bearing that wasn't horizontal (so that the direction isn't necessarily to be taken as on the ground).

Then we have the statement "it passes to the north of O". This seems to mean that the particle heads off to the West of B, and hence that the angle of 60^0 is being measured between the (horizontal) direction and the line BO (ie from B to O).

Q12 The expected number of throws in (i) (not given in the official sol'ns) is $\frac{1+2q}{1-q^2}$

Q13 I wonder if the following argument is acceptable:

Let r = k(k-1) & n = 730

Then 1 - $e^{-k(k-1)/730} \approx 1 - \frac{n-r}{n} = \frac{r}{n} = \frac{{}^{k}C_2}{365}$

Prob(at least 2 guests share the same birthday) =

Prob(guests 1&2 share the same birthday) + [other combinations]

$\approx \frac{1}{365} \cdot {}^{k}C_2$ [treating the probability of overlap as sufficiently small]

Q14 The 'refinement' here was justifying the section of the pdf that the median (m) fell into. One way of doing this is just to make the assumption that k<m<2k, do the integration, and then confirm that the resulting value of m lies between k & 2k.