STEP 2005, Paper 1, Q1 Solution (2 pages; 9/5/18)

(i) A 'case by case' approach can be adopted here. We could, for example, consider the different possibilities for the 1st digit:

79999: 1 rearrangement of 9999

89998: 4 rearrangements of 9998

99997: 4 rearrangements of 9997

99988: ${}^4C_2 = 6$ rearrangements of 9988

giving a total of 15

Alternatively, we could categorise the numbers according to the number of 9s:

4 9s: 99997: 5 rearrangements

3 9s: 99988: $\frac{5!}{3!2!} = 10$ rearrangements

again, giving a total of 15

(ii) The 2nd approach in (i) seems to be shorter:

4 9s: 99993: 5 rearrangements

3 9s: 99984/99975/99966:

$\frac{5!}{3!} + \frac{5!}{3!} + \frac{5!}{3!2!} = 20 + 20 + 10 = 50$ rearrangements

2 9s: 99885/99876/99777 (we can consider the number of 8s here)

$\frac{5!}{2!2!} + \frac{5!}{2!} + \frac{5!}{2!3!} = 30 + 60 + 10 = 100$ rearrangements

1 9: 98886/98877 (again, considering the number of 8s)

$\frac{5!}{3!} + \frac{5!}{2!2!} = 20 + 30 = 50$ rearrangements

0 9s: 88887: 5 rearrangements

giving a total of $5 + 50 + 100 + 50 + 5 = 210$ rearrangements