Roots of Polynomial Equations - Exercises

(2 pages; 14/01/20)

Key to difficulty:

* introductory exercise

** light A Level (FM) standard

*** harder A Level (FM) standard

background - light exercise

background - harder exercise

(1#) If the quadratic equation $2x^2 + 5x - 9 = 0$ has roots α and β, find the quadratic equation which has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

(2#) If the roots of the equation $x^2 + x - 13 = 0$ are α & β, find the equation with roots $2\alpha + 3\beta$ & $3\alpha + 2\beta$

(3#) If the roots of the equation $x^3 - 14x^2 + 56x - 64 = 0$ are α, β & γ, find the equation with roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ & $\frac{1}{\gamma}$

(4***) Find the roots of the equation $x^3 - 14x^2 + 56x - 64 = 0$, given that they form a geometric progression.

(5***) If the roots of the equation $x^5 + bx^4 + cx^3 + \mathrm{d}x^2 + ex + f = 0$ are 5 consecutive positive integers, find expressions for these roots.

(6#) If α, β and γ are the roots of the equation

$x^3 - 14x^2 + 56x - 64 = 0$,

find the equation with roots $\alpha\beta$, $\alpha\gamma$ and $\beta\gamma$.

(7##) If α, β and γ are the roots of the equation

$x^3 - 2x^2 - 4x + 5 = 0$,

find the equation with roots $\alpha + \beta\gamma$, $\beta + \alpha\gamma$ and $\gamma + \alpha\beta$.