Pure - Miscellaneous: Exercises (3 pages; 17/1/20)

Contents

Key to difficulty:

* introductory exercise

** light A Level (FM) standard

*** harder A Level (FM) standard

\# background - light exercise

\#\# background - harder exercise

(A) Indices

(1*) (i) Does $\sqrt{4}$ equal 2 or ± 2? (ii) Simplify $\sqrt{x^2}$

(2*) Simplify $\left(1 + \left(1 + 2^{-\frac{1}{2}} \right)^{-1} \right)^{-1}$

(B) Partial Fractions

(1***) Express $\frac{1}{(1-x^2)^2}$ in terms of partial fractions

(C) Recurrence relations

(1#) Consider the sequence defined by $u_n = au_{n-1} + b$,

where a & b are real constants, and u_0 is given.

(i) What familiar sequences are special cases of this sequence?

(ii) Define a new sequence by $v_n = u_n + c$

For what value of c, in terms of a & b, will v_n be a geometric sequence?

For what value of a does this not work?

(iii) If $u_n = 2u_{n-1} + 3$, and $u_0 = 4$, find a formula for u_n in terms of n

(iv) Find a similar formula for $u_n = au_{n-1} + b$, where u_0 is given.

(v) Under what conditions will u_n be constant? Give a non-trivial example.

(Z) Miscellaneous

(1*) How are the following usually defined?

(a) Whole numbers (b) Natural numbers

(2#) Prove that $E' \Rightarrow L'$ is equivalent to $L \Rightarrow E$

(3#) What is a transcendental number?

(4#) Find the square roots of $49 - 12\sqrt{5}$

(5#) Show that $\sum_{r=0}^{n} \binom{n}{r} = 2^n$