

Pure - Miscellaneous: Exercises (2 pages; 7/10/18)**Contents**

- (A) Indices
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(A) Indices

(1) (i) Does $\sqrt{4}$ equal 2 or ± 2 ? (ii) Simplify $\sqrt{x^2}$

(2) Simplify $\left(1 + \left(1 + 2^{-\frac{1}{2}}\right)^{-1}\right)^{-1}$

(B) Partial Fractions

(1) Express $\frac{1}{(1-x^2)^2}$ in terms of partial fractions

(C) Recurrence relations

(1) Consider the sequence defined by $u_n = au_{n-1} + b$,

where a & b are real constants, and u_0 is given.

(i) What familiar sequences are special cases of this sequence?

(ii) Define a new sequence by $v_n = u_n + c$

For what value of c , in terms of a & b , will v_n be a geometric sequence?

For what value of a does this not work?

(iii) If $u_n = 2u_{n-1} + 3$, and $u_0 = 4$, find a formula for u_n in terms of n

(iv) Find a similar formula for $u_n = au_{n-1} + b$, where u_0 is given.

(v) Under what conditions will u_n be constant? Give a non-trivial example.

(Z) Miscellaneous

(1) How are the following usually defined?

(a) Whole numbers (b) Natural numbers

(2) Prove that $E' \Rightarrow L'$ is equivalent to $L \Rightarrow E$

(3) What is a transcendental number?

(4) Find the square roots of $49 - 12\sqrt{5}$

(5) Show that $\sum_{r=0}^n \binom{n}{r} = 2^n$