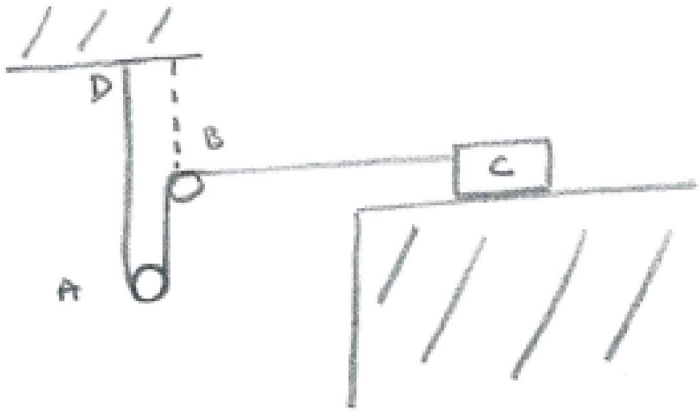
Pulley Exercises (Solutions) (5 pages; 11/3/17)

(1)



Referring to the diagram, A is a smooth pulley of mass 2 kg, which can move up and down; B is a smooth, fixed pulley, and C is a block of mass 1kg, which is initially held at rest on a table. A light inextensible rope is fixed at D, and leads to C, via the two pulleys.

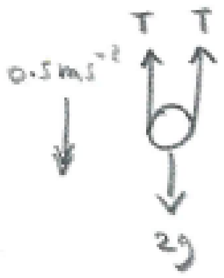
C is now released and accelerates at 2 ms^{-1}. Find the coefficient of friction, μ between C and the table.

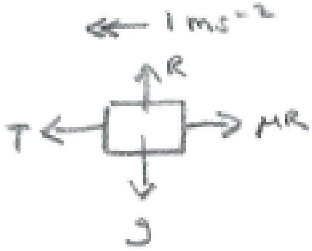
Solution

First of all, the tension (T, say) is the same throughout the rope (since the rope is light and the pulleys are smooth - see note (i) below).

Also, because A falls by half the distance that C moves, its acceleration is also half that of C - see note (ii) below).

[The fact that the rope is inextensible ensures that the rope (as a whole) and the block move the same distances.]





From the force diagram for A,

N2L $\Rightarrow 2g - 2T = (2)(1) \Rightarrow T = g - 1$ [1]

From the force diagram for B,

N2L $\Rightarrow T - \mu R = (1)(2)$

Also, vertical equilibrium $\Rightarrow R = g$,

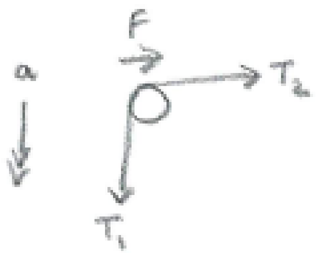
so that $T - \mu g = 2$ [2]

Then [1]&[2] $\Rightarrow \mu g = g - 3$,

so that $\mu = 1 - \frac{3}{9.8} = 0.694$ (3sf)

Notes

(i) Referring to the force diagram for the rope around B, for example:



Suppose that the rope has mass m, that the pulley exerts a frictional force F, and that the rope experiences forces T_1 and T_2 at its ends.

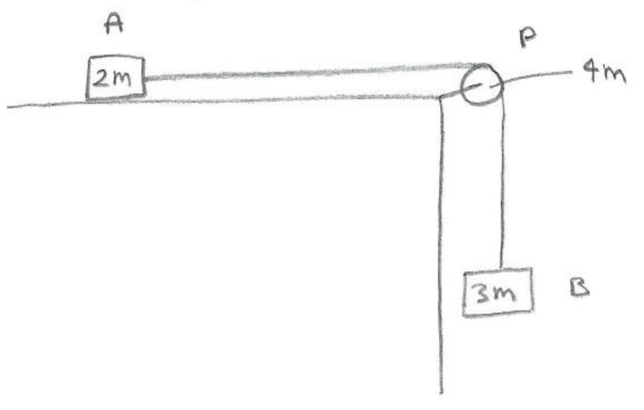
If the rope has acceleration a,

then $T_1 - T_2 - F = ma$

If $F = 0$ (ie if the pulley is smooth), and $m \approx 0$ (ie the rope is 'light', and so has negligible mass), then $T_1 - T_2 \approx 0$, and so the tensions are approximately equal.

(ii) From the suvat equation, if $u = 0$, then $s = \frac{1}{2}at^2$, so that the acceleration is proportional to the distance, for a given t.

(2) Rotating Pulley



Initially block A is held at rest on a smooth table. The pulley P can rotate freely. The string leading from A to B, passing over P, is light and inextensible.

The pulley is a uniform disc of radius r, and the blocks can be modelled as particles.

Block A is released. The tension in the section of the string AP is T_A and in PB it is T_B.

Assuming that the string does not slip on the pulley, and that A does not reach P,

(i) Show that the angular acceleration of the pulley is $\frac{3g}{7r}\ rads^{-2}$

(ii) Find T_A and T_B in terms of m and g.

Solution

The moment of inertia, I of P is $\frac{1}{2}(4m)r^2 = 2mr^2$ [standard result for a disc about its axis]

The total moment of the external forces on P about its axis, $C = I\ddot{\theta}$, where $\ddot{\theta}$ is the angular acceleration of P.

$C = T_B r - T_A r$

Hence $r(T_B - T_A) = 2mr^2\ddot{\theta}$ (1)

The acceleration of A and B is $r\ddot{\theta}$ [the distance fallen by $B = r\theta$ (the arc length travelled by a point on the circumference of the pulley), and this is differentiated twice]

So, for A, N2L $\Rightarrow T_A = (2m)(r\ddot{\theta})$ (2)

and for B: $(3m)g - T_B = (3m)(r\ddot{\theta})$ (3)

Substituting for T_A and T_B from (2) & (3) into (1):

$(3mg - 3mr\ddot{\theta}) - 2mr\ddot{\theta} = 2mr\ddot{\theta}$

so that $7mr\ddot{\theta} = 3mg$, and $\ddot{\theta} = \frac{3g}{7r} \quad rads^{-2}$

Alternative method

By Conservation of energy,

$\frac{1}{2}I(\dot{\theta})^2 + \frac{1}{2}(2m)(r\dot{\theta})^2 + \frac{1}{2}(3m)(r\dot{\theta})^2 - (3m)g(r\theta) = \text{constant}$

(taking the initial position of B as the zero of PE; as before, $r\theta$ is the distance that B has fallen when P has rotated by θ rad)

Hence $\left\{\left(\frac{1}{2}\right)2mr^2 + \left(\frac{5}{2}\right)mr^2\right\}(\dot{\theta})^2 - 3mgr\theta = \text{constant}$

Differentiating wrt time,

$\left(\frac{7}{2}\right)mr^2(2)\dot{\theta}\ddot{\theta} - 3mgr\dot{\theta} = 0$,

so that $7r\ddot{\theta} - 3g = 0$, and $\ddot{\theta} = \frac{3g}{7r} \quad rads^{-2}$

(ii) From (2), $T_A = 2mr\left(\frac{3g}{7r}\right) = \frac{6mg}{7}$

From (3), $T_B = 3mg - 3mr\left(\frac{3g}{7r}\right) = \frac{12mg}{7}$