Proof – Q2 [Practice/E](8/7/21)

Prove that the sum of the squares of consecutive positive integers is odd.

Solution

Case 1: The smaller integer is even; say $2n$

Then $(2n)^2 + (2n+1)^2 = 4n^2 + 4n^2 + 4n + 1$

$= 2(4n^2 + 2n) + 1$, which is odd.

Case 2: The smaller integer is odd; say $2n+1$

Then $(2n+1)^2 + (2n+2)^2 = 4n^2 + 4n + 1 + 4n^2 + 8n + 4$

$= 2(4n^2 + 6n + 2) + 1$, which is odd.