Proof Overview (4/10/21; 3 pages)

Q1 [Practice/E]

If n is a positive integer, and n^2 is odd, prove that n is odd.

Q2 [Practice/E]

Prove that the sum of the squares of consecutive positive integers is odd.

Q3 [Practice/M]

Prove that there are no positive integers m and n such that

$m^2 = n^2 + 1$

Q4 [Problem/E]

Prove that $E' \Rightarrow L'$ is equivalent to $L \Rightarrow E$

Q5 [Problem/E]

Suppose that a half price offer applies at selected stores of a supermarket for customers with loyalty cards.

H is "Half price offer applies"

S is "Customer shops at a selected store"

L is "Customer has a loyalty card"

Place the following statements into equivalent groups. Which ones are true?

$H \Rightarrow S$

$H \Leftarrow S$

"H is a necessary condition for S"

"S is a necessary condition for H"

"H is a sufficient condition for S"

"S is a sufficient condition for H"

"H is only true if S is true"

"S is only true if H is true"

Q6 [Problem/E]

Let A be "$x = 3$" , and let B be "$x^2 = 9$ "

Which of the following statements are true?

A is a necessary but not sufficient condition for B

A is a sufficient but not necessary condition for B

B is a necessary but not sufficient condition for A

B is a sufficient but not necessary condition for A

A (is true) only if B (is true)

B (is true) only if A (is true)

Q7 [Problem/E]

For the following statements, group together the ones that are equivalent.

A: $X \Rightarrow Y$

B: Y is a sufficient condition for X

C: X is a necessary condition for Y

D: X is true only if Y is true

E: Y is true if X is true

F: If Y isn't true then X isn't true

G: If Y is true, then X is true