Polynomials – Q5 (26/6/23)

If the roots of the equation $x^5 + bx^4 + cx^3 + \mathrm{d}x^2 + ex + f = 0$ are 5 consecutive positive integers, find expressions for these roots.

Solution

Let the roots be $\alpha - 2,\ \alpha - 1,\ \alpha,\ \alpha + 1$ & $\alpha + 2$

Then, summing these, $5\alpha = -b$

and hence the roots are $-(\frac{b}{5} + 2)$, $-(\frac{b}{5} + 1)$, $-\frac{b}{5}$, $1 - \frac{b}{5}$ & $2 - \frac{b}{5}$