Polar Curves – Q3 [8 marks](12/6/21)

Exam Boards

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 2)

AQA: Pure (Year 2)

Edx: Core Pure (Year 2)

Convert the curve $r = \dfrac{2}{1+\cos\theta}$ to cartesian form, and sketch the curve, based on its cartesian form. [8 marks]

Solution

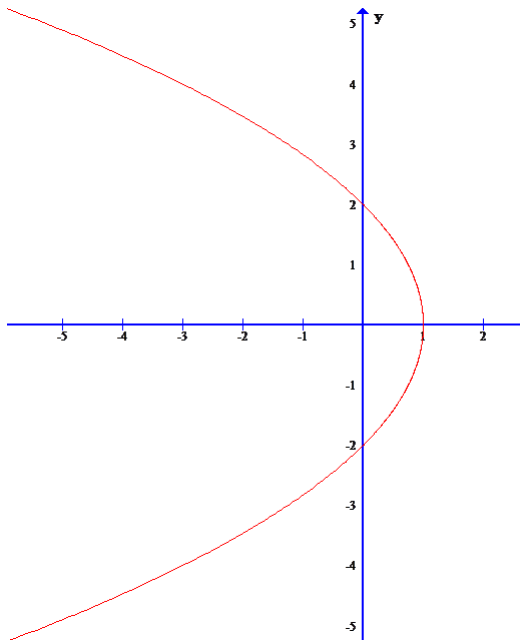$r = \dfrac{2}{1+\cos\theta}$; $x = r\cos\theta$ and $y = r\sin\theta$ [1 mark]

Also $r^2 = x^2 + y^2$

So $r + r\cos\theta = 2 \Rightarrow r = 2 - x \Rightarrow r^2 = (2 - x)^2$ [1 mark]

$\Rightarrow x^2 + y^2 = 4 + x^2 - 4x \Rightarrow y^2 = 4(1 - x)$ [2 marks]

This can be obtained from the parabola $y^2 = 4x$ by the following steps:

$y^2 = 4(-x) = -4x$ [reflection in the y-axis; note that the curve now only exists for negative x] [1 mark]

$y^2 = -4(x - 1) = 4(1 - x)$ [translation of $\begin{pmatrix}1\\0\end{pmatrix}$] [1 mark]

[2 marks]