Polar Curves – Q1 [13 marks](12/6/21)

Exam Boards

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 2)

AQA: Pure (Year 2)

Edx: Core Pure (Year 2)

(i) Sketch the curve $r = 5 + 4cos\theta$, explaining how you obtained your sketch. [Do not obtain your curve entirely by plotting points.] [5 marks]

(ii) Without converting the curve to cartesian form, find the greatest negative x-coordinate of a point on the curve. [4 marks]

(iii) Determine the area enclosed by the curve. [4 marks]

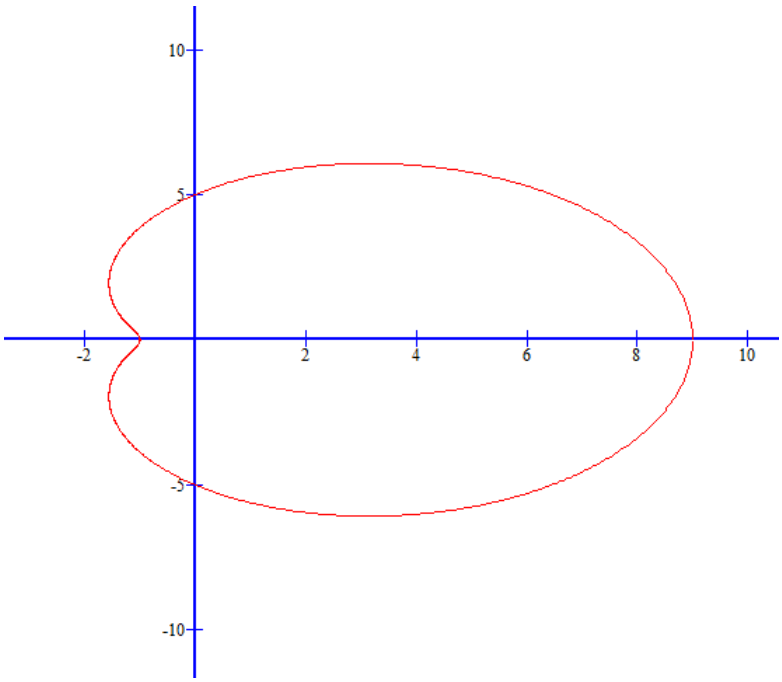
Solution

(i) $r = 5 + 4cos\theta$

Step 1: As r is a function of $cos\theta$, the curve will be symmetric about the x-axis. [1 mark]

Step 2: $r > 0$ at all times [1 mark]

Step 3: Key points to plot are $\theta = 0, r = 9;\ \theta = \frac{\pi}{2}, r = 5;\ \theta = \pi, r = 1$



[3 marks]

(ii) The required x-coordinate can be found by investigating the vertical tangents; ie when $\frac{dx}{d\theta} = 0$ [when the x-coordinate is (instantaneously) not changing as θ changes] [1 mark]

$x = rcos\theta = (5 + 4\cos\theta)cos\theta$

so that $\frac{dx}{d\theta} = (-4sin\theta)cos\theta + (5 + 4cos\theta)(-sin\theta) =$
$-8sin\theta cos\theta - 5sin\theta$ [1 mark]

Then $\frac{dx}{d\theta} = 0 \Rightarrow sin\theta = 0$ (ie $\theta = 0\ or\ \pi$) or $cos\theta = -\frac{5}{8}$ [1 mark]

$\Rightarrow x = (5 + 4\cos\theta)cos\theta = \left(5 - \frac{20}{8}\right)\left(-\frac{5}{8}\right) = -\frac{25}{16}$ [1 mark]

(iii) Area enclosed by curve $= 2\int_0^{\pi} \frac{1}{2}(5 + 4\cos\theta)^2 d\theta$ [1 mark]

$= \int_0^{\pi} 25 + 16cos^2\theta + 40cos\theta\ d\theta$

$= \int_0^{\pi} 25 + 8(1 + cos2\theta) + 40cos\theta\ d\theta$ [1 mark]

$= [33\theta + 4sin2\theta + 40sin\theta]_0^{\pi}$ [1 mark]

$= 33\pi$ [1 mark]

[Rough check: Area of rectangle of base 11 and height 10 is approx. 35π]