Parametric Equations – Q1 [Problem/H](30/5/21)

Express the following parametric equations in Cartesian form (ie a relation between x & y).

(i) $x = 2t + t^2,\ y = 2t^2 + t^3$ [3 marks]

(ii) $x = 5t^2 - 4,\ y = 9t - t^3$ [5 marks]

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Solution

(i) $x = 2t + t^2,\ y = 2t^2 + t^3 \Rightarrow x = t(2 + t), y = t^2(2 + t)$

So $\frac{y}{x} = t$

Then $x = 2\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2$, and hence $x^3 = 2xy + y^2$

(ii) $x = 5t^2 - 4,\ y = 9t - t^3 = t(9 - t^2)$; then $t^2 = \frac{x+4}{5}$,

and also $\frac{y}{t} - 9 = -t^2$; so $\frac{x+4}{5} = 9 - \frac{y}{t}$,

and hence $\frac{y}{t} = 9 - \frac{x+4}{5} = \frac{45-x-4}{5} = \frac{41-x}{5}$,

so that $t = \frac{5y}{41-x}$

Then , substituting back into $x = 5t^2 - 4$, we have

$x = 5\left(\frac{5y}{41-x}\right)^2 - 4$,

and hence $(x + 4)(41 - x)^2 = 125y^2$