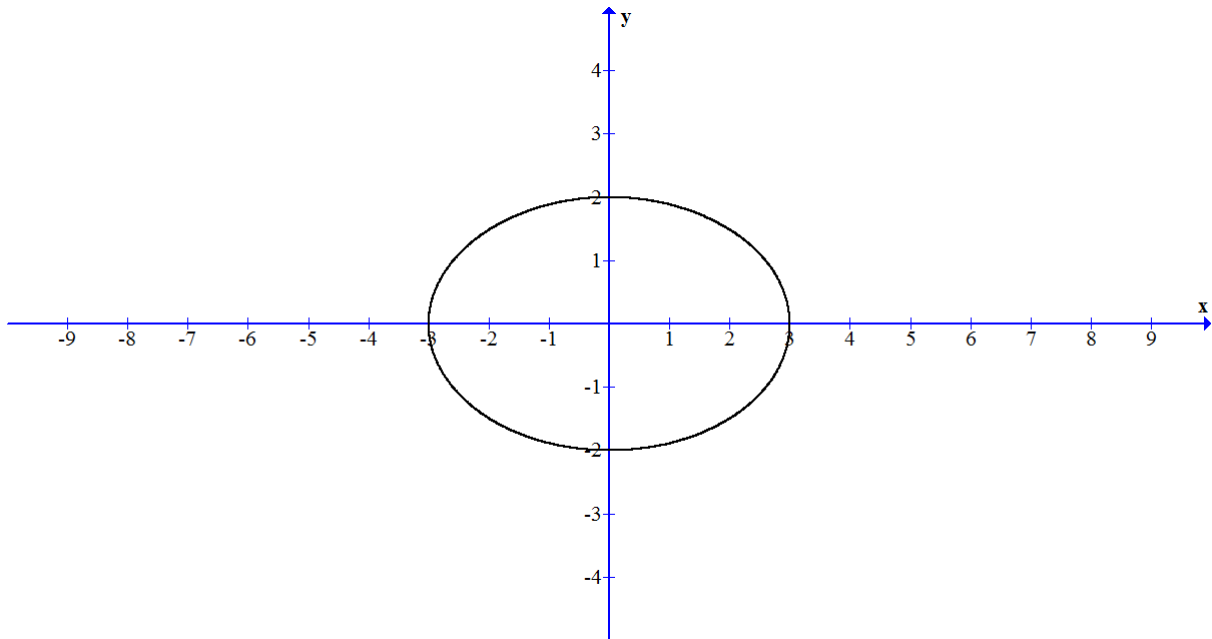


Parametric Equations - Examples (30/8/2013)

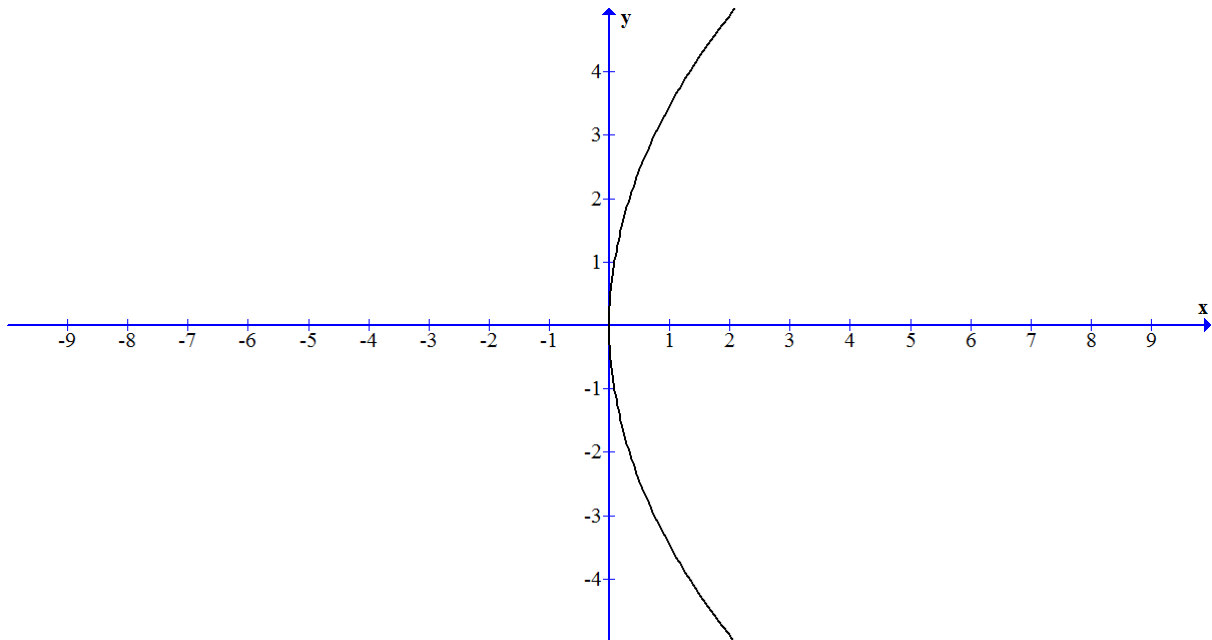
(i) $x = 3\cos\theta$, $y = 2\sin\theta$

Cartesian form: $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ (ellipse)



(ii) $x = 3t^2$, $y = 6t$

Cartesian form: $y^2 = 12x = 4(3)x$; parabola with focus at $(3,0)$



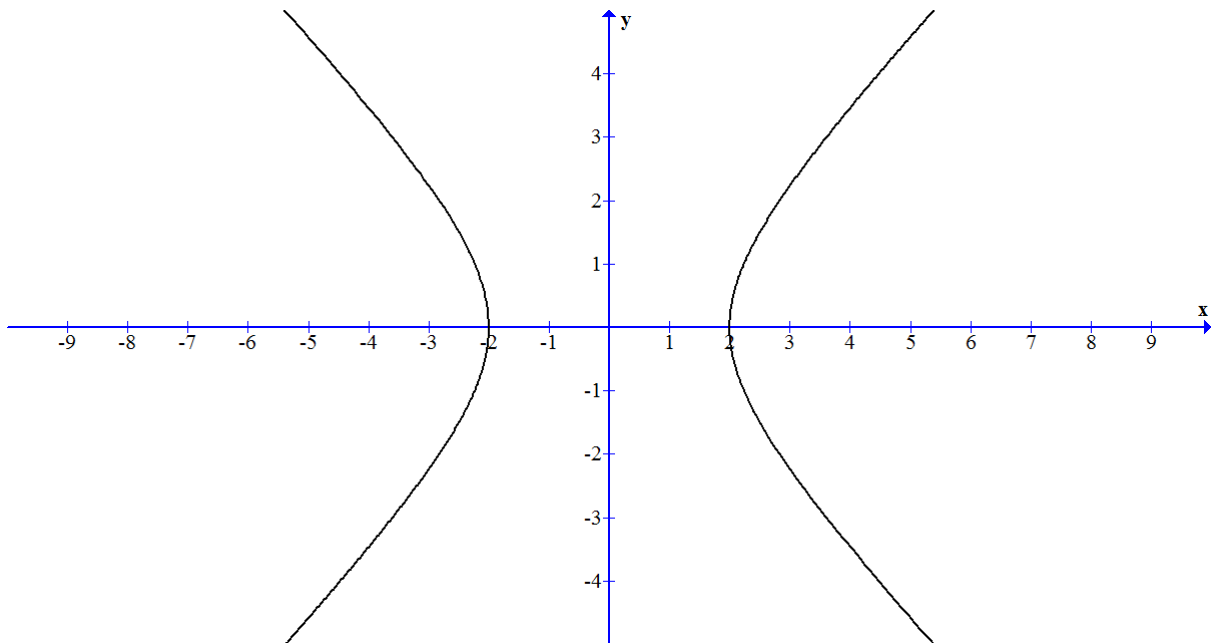
$$(iii) \quad x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}$$

Cartesian form:

$$x + y = 2t; \quad x - y = \frac{2}{t}$$

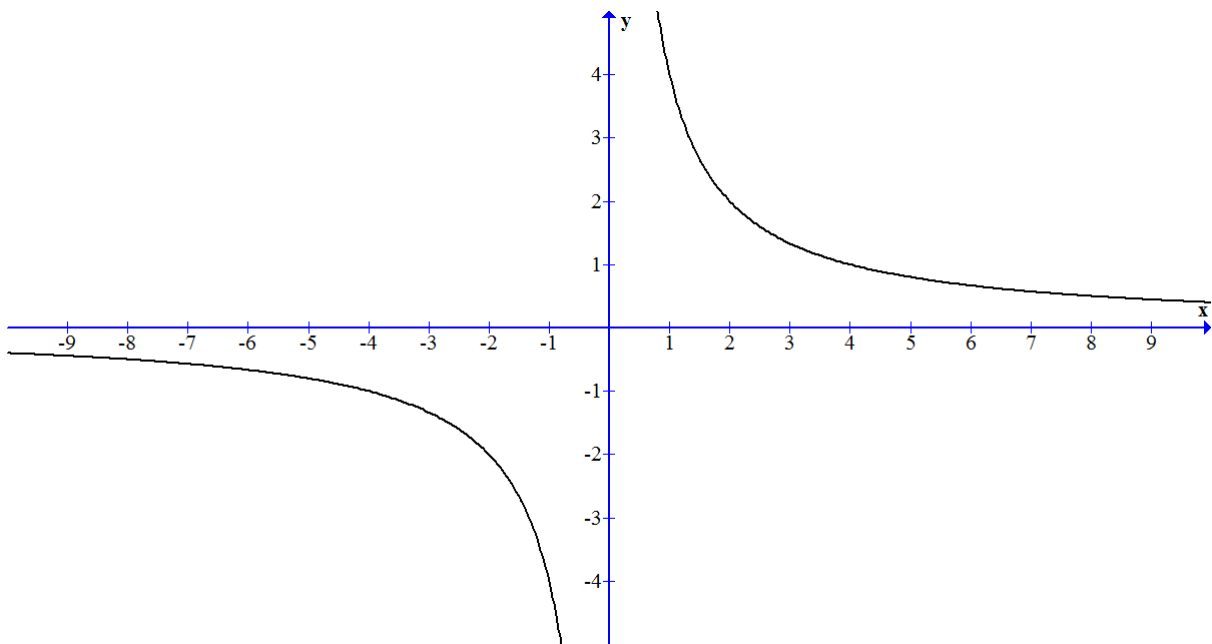
$$\Rightarrow (x + y)(x - y) = 4$$

$$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{2^2} = 1 \quad (\text{rectangular hyperbola})$$



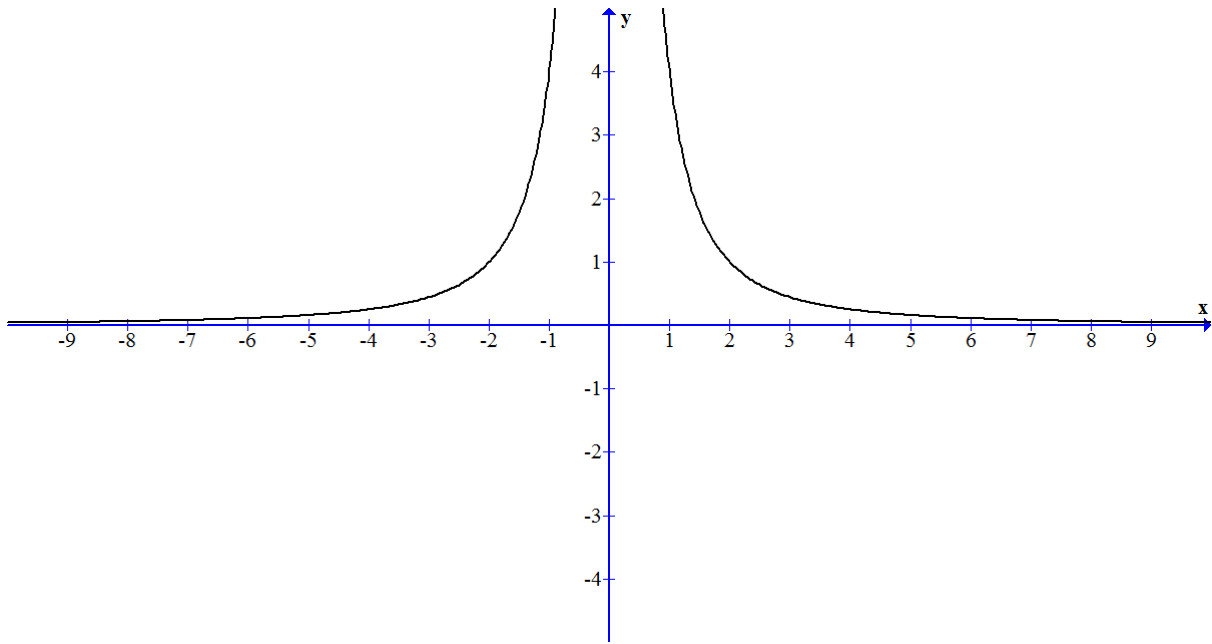
$$(iv) x = 2t, y = \frac{2}{t}$$

Cartesian form: $xy = 4$ (rectangular hyperbola, with asymptotes being x and y axes)



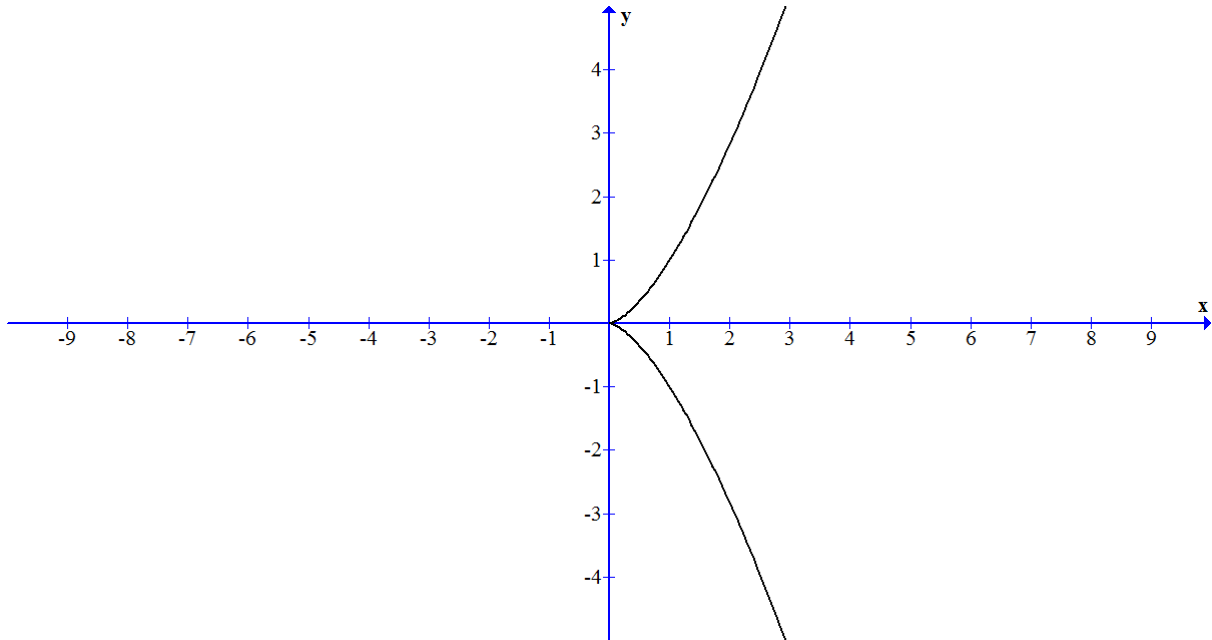
$$(v) \ x = 2t, \ y = \frac{1}{t^2}$$

$$\text{Cartesian form: } x^2 = \frac{4}{y} \Rightarrow y = \frac{4}{x^2}$$



$$(vi) x = t^2, y = t^3$$

$$\text{Cartesian form: } x^3 = y^2$$

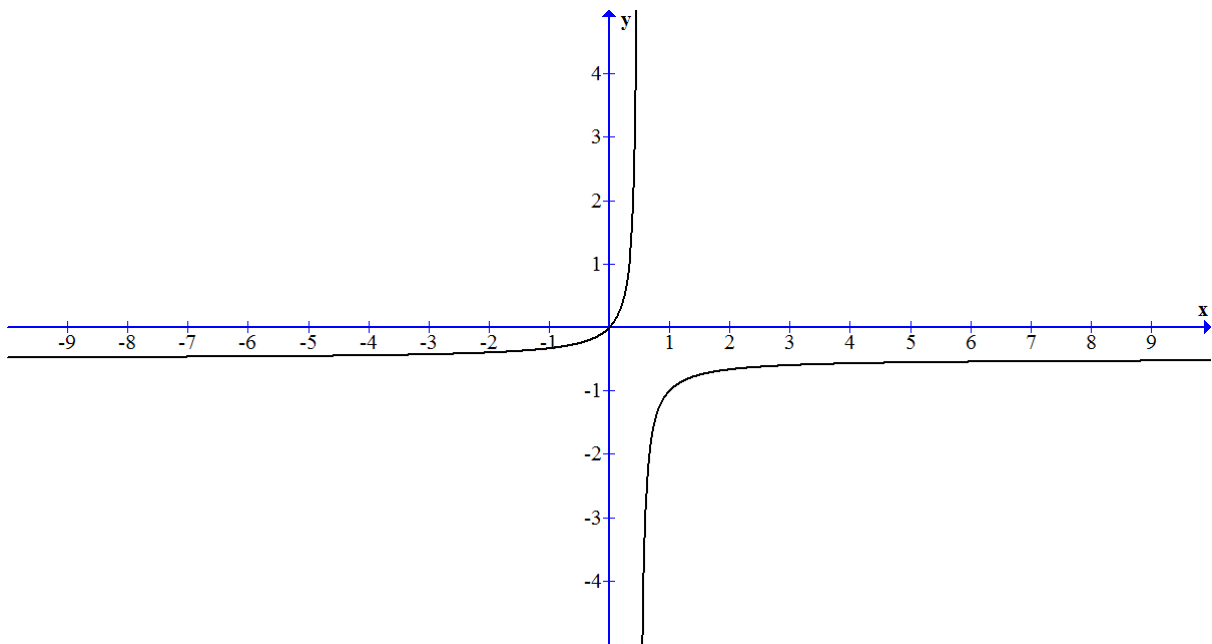


$$(vii) \ x = \frac{t}{1+t}, \ y = \frac{t}{1-t}$$

$$\text{Cartesian form: } xy = \frac{t^2}{1-t^2}; \ y - x = \frac{2t^2}{1-t^2}$$

$$\Rightarrow y - x = 2xy \Rightarrow y(1 - 2x) = x$$

$$\Rightarrow y = \frac{x}{1-2x}$$



$$\text{(viii) } x = \frac{t}{3-t}, \quad y = \frac{t^2}{3-t}$$

$$\text{Cartesian form: } \frac{y}{x} = t \Rightarrow x = \frac{\left(\frac{y}{x}\right)}{3-\frac{y}{x}} = \frac{y}{3x-y}$$

$$\Rightarrow 3x^2 - xy = y \Rightarrow y(1+x) = 3x^2$$

$$\Rightarrow y = \frac{3x^2}{1+x}$$

