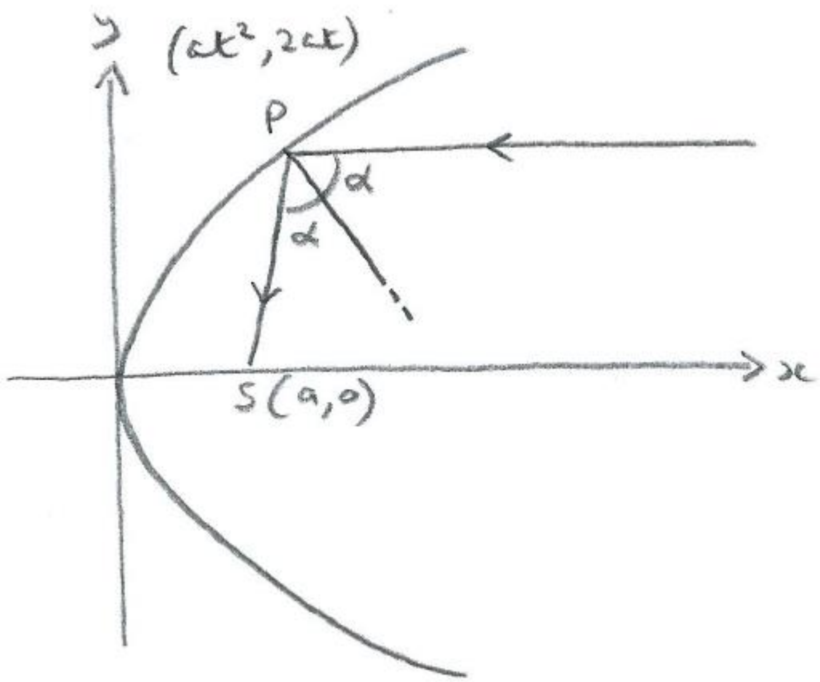
Parabolas - Exercises (2 pages; 18/8/19)

See also the separate note "Parabolas" for further exercises.

(1) Using the parametric equations of a parabola ($x = at^2$, $y = 2at$), show that the midpoints of chords of a parabola that have the same direction lie on a straight line parallel to the x-axis.

[A chord of a parabola joins two points on the parabola.]

(2) A ray (eg of light) travels on a path parallel to the x-axis and hits the surface of the parabola $y^2 = 4ax$ at the point P ($at^2, 2at$). The angle between the incoming ray and the normal at P is α. It can be assumed that the angle that the reflected ray makes with the normal is also α .



(i) Show that $tan\alpha = t$

(ii) Find the gradient of the reflected ray.

(iii) Show that the reflected ray passes through the focus of the parabola.

(3) Suppose that $P\ (ap^2, 2ap)$ and $Q\ (aq^2, 2aq)$ are two points on the parabola $y^2 = 4ax$, such that the chord PQ passes through the focus of the parabola. Show that $pq = -1$.

(4) If the tangents to a parabola at P and Q are perpendicular, show that the chord PQ passes through the focus S of the parabola.

(5) Find the cartesian equations of the parabolas with:

(i) focus (4,4) and directrix $y = 0$

(ii) focus (2,2) and directrix $x + y + 2 = 0$

(6) Suppose that $P\ (ap^2, 2ap)$ and $Q\ (aq^2, 2aq)$ are two points on the parabola $y^2 = 4ax$, such that the chord PQ passes through the focus of the parabola. Show that the tangents at P and Q meet on the directrix.

(7) $P\ (ap^2, 2ap)$ and $Q\ (aq^2, 2aq)$ are two points on the parabola $y^2 = 4ax$, such that the chord PQ passes through the focus of the parabola. Show that the locus of the midpoint of PQ is a parabola, and establish its focus and directrix.