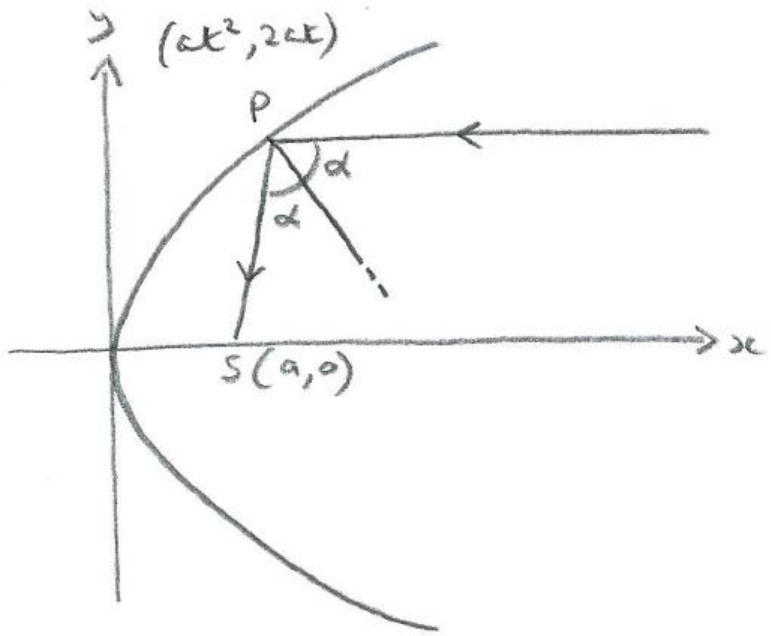
Parabolas Q2 [Problem/H] (29/5/21)

A ray (eg of light) travels on a path parallel to the x-axis and hits the surface of the parabola $y^2 = 4ax$ at the point P $(at^2, 2at)$. The angle between the incoming ray and the normal at P is α. It can be assumed that the angle that the reflected ray makes with the normal is also α .

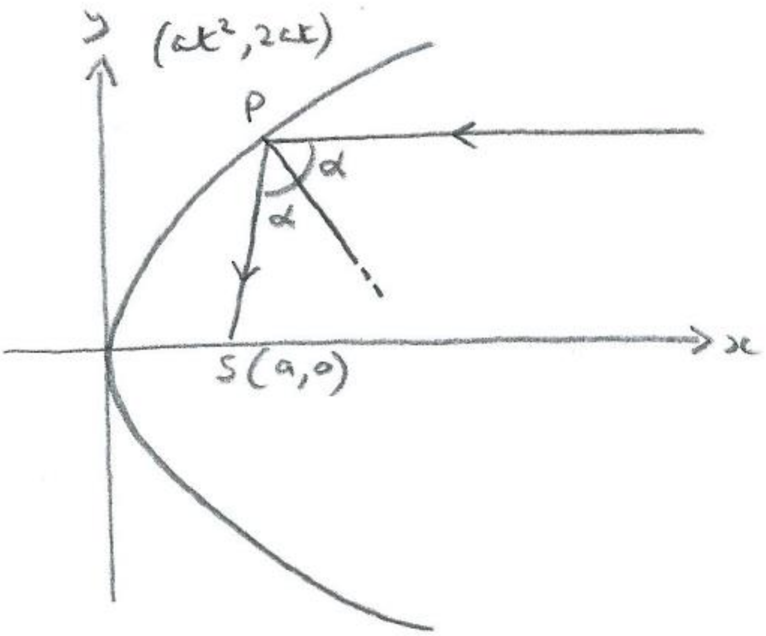


(i) Show that $tan\alpha = t$

(ii) Find the gradient of the reflected ray.

(iii) Show that the reflected ray passes through the focus of the parabola.

A ray (eg of light) travels on a path parallel to the x-axis and hits the surface of the parabola $y^2 = 4ax$ at the point P $(at^2, 2at)$. The angle between the incoming ray and the normal at P is α. It can be assumed that the angle that the reflected ray makes with the normal is also α .



(i) Show that $tan\alpha = t$

(ii) Find the gradient of the reflected ray.

(iii) Show that the reflected ray passes through the focus of the parabola.

Solution

(i) The gradient of the tangent at P is $\frac{1}{t}$ (standard result), and hence the gradient of the normal is $-t$.

Hence, as the normal makes an angle $\pi - \alpha$ with the positive x-axis, $-t = tan(\pi - \alpha) = -tan\alpha$, so that $tan\alpha = t$.

(ii) The gradient of the reflected ray is $\tan(\pi - 2\alpha) = -\tan\,(2\alpha)$

$= \frac{-2tan\alpha}{1-tan^2\alpha} = \frac{2t}{t^2-1}$

(iii) The equation of the reflected ray is $y - 2at = \frac{2t}{t^2-1}(x - at^2)$.

When it meets the x-axis, $-2at = \frac{2t}{t^2-1}(x - at^2)$,

and $-a(t^2 - 1) = x - at^2$,

so that $x = a$; ie the reflected ray passes through the focus.