Parabolas Overview (30/5/21)

Q1 [4 marks]

Using the parametric equations of a parabola ($x = at^2$, $y = 2at$), show that the midpoints of chords of a parabola that have the same direction lie on a straight line parallel to the x-axis.

[A chord of a parabola joins two points on the parabola.]

Q2 [Problem/H]

A ray (eg of light) travels on a path parallel to the x-axis and hits the surface of the parabola $y^2 = 4ax$ at the point P $(at^2, 2at)$. The angle between the incoming ray and the normal at P is α. It can be assumed that the angle that the reflected ray makes with the normal is also α .

Q3 [15 marks]

Suppose that $P\ (ap^2, 2ap)$ and $Q\ (aq^2, 2aq)$ are two points on the parabola $y^2 = 4ax$, such that the chord PQ passes through the focus of the parabola.

(i) Show that $pq = -1$. [7 marks]

(ii) Show that the tangents at P and Q meet on the directrix.

[The equations of the tangents can be quoted without proof.]

[3 marks]

(iii) Show that the locus of the midpoint of PQ is a parabola, and establish its focus and directrix. [5 marks]

Q4 [5 marks]

If the tangents to a parabola at P and Q are perpendicular, show that the chord PQ passes through the focus S of the parabola.

[The equation of the tangent can be used without proof.]

Q5 [Problem/H]

Find the cartesian equations of the parabolas with:

(i) focus (4,4) and directrix $y = 0$

(ii) focus (1,1) and directrix $x + y + 2 = 0$