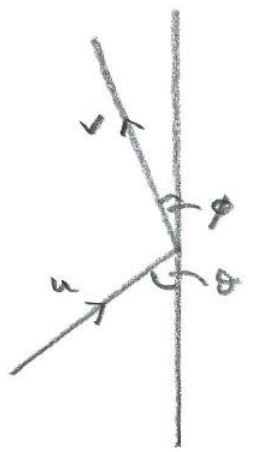
Oblique Impact with Plane - Exercises (2/3/19)



Referring to the diagram above,

(1) Find an expression for $tan\phi$ in terms of $tan\theta$ and e.

(2) Find an expression for v in terms of $u, \theta \ and \ e$

(i) involving $cos\theta \ and \ sin\theta$

(ii) involving $tan\theta$

(3) When $\theta = 60°$ and $e = \frac{1}{\sqrt{3}}$, find ϕ, and v (in terms of u).

(4) What relation must hold between $tan\theta$ and e, in order for the outgoing path to be perpendicular to the incoming path?

(5) For the same situation, express v in terms of u and e.

(6) For the same situation, what is the smallest possible value for θ?

Solutions

(1) $vcos\phi = ucos\theta$ (A) and $vsin\phi = esin\theta$ (B)

Dividing (B) by (A), $etan\theta = tan\phi$

(2)(i) $(A)^2 + (B)^2 \Rightarrow v^2(cos^2\phi + sin^2\phi) = u^2(cos^2\theta + e^2sin^2\theta)$,

so that $v = u\sqrt{cos^2\theta + e^2sin^2\theta}$

(ii) $v = u\sqrt{cos^2\theta + e^2sin^2\theta} = u\sqrt{\frac{1+e^2tan^2\theta}{sec^2\theta}}$

$= u\sqrt{\frac{1+e^2tan^2\theta}{1+tan^2\theta}}$

(3) When $\theta = 60°$ and $e = \frac{1}{\sqrt{3}}$,

$tan\phi = etan\theta = \frac{1}{\sqrt{3}}.\sqrt{3} = 1$, so that $\phi = 45°$

And $v = u\sqrt{\frac{1+e^2tan^2\theta}{1+tan^2\theta}} = u\sqrt{\frac{1+tan^2\phi}{1+tan^2\theta}} = u\sqrt{\frac{1+1}{1+3}} = \frac{u}{\sqrt{2}}$

(4) If $\theta + \phi = 90°$, $tan\phi = etan\theta$ and $tan\phi = \tan(90° - \theta)$

$= cot\theta = \frac{1}{tan\theta}$

Hence $etan\theta = \frac{1}{tan\theta}$, so that $tan^2\theta = \frac{1}{e}$ and $tan\theta = \frac{1}{\sqrt{e}}$

(5) $v = u\sqrt{\frac{1+e^2tan^2\theta}{1+tan^2\theta}} = u\sqrt{\frac{1+e^2(\frac{1}{e})}{1+\frac{1}{e}}} = u\sqrt{\frac{e+e^2}{e+1}} = u\sqrt{e}$

(6) $tan\theta$, and hence θ, is minimised when e is maximised; ie when $e = 1$ and $tan\theta = 1$, so that $\theta = 45°$