

Numerical Methods – Q4: Integration [Practice/M]
(12/6/21)

$\int_0^{\frac{\pi}{2}} \sqrt{\sin x} dx$ is to be estimated. Complete the following table, by the quickest method. Give values to 6 dp.

n	T_n	M_n	S_n
1			
2			
4			
8			

Solution

Let $f(x) = \sqrt{\sin x}$

$$T_1 = \frac{1}{2} \left(\frac{\pi}{2} \right) \left(f(0) + f \left(\frac{\pi}{2} \right) \right) = \frac{\pi}{4} (0 + 1) = 0.785398$$

$$M_1 = \left(\frac{\pi}{2} \right) f \left(\frac{\pi}{4} \right) = \left(\frac{\pi}{2} \right) 0.840896 = 1.320876$$

$$S_2 = \frac{2}{3} M_1 + \frac{1}{3} T_1 = 1.142383$$

$$T_2 = \frac{T_1 + M_1}{2} = 1.053137$$

$$M_2 = \left(\frac{\pi}{4} \right) \left(f \left(\frac{\pi}{8} \right) + f \left(\frac{3\pi}{8} \right) \right) = \left(\frac{\pi}{4} \right) (0.618614 + 0.961187) \\ = 1.240773$$

$$S_4 = \frac{2}{3} M_2 + \frac{1}{3} T_2 = 1.178228$$

$$T_4 = \frac{T_2 + M_2}{2} = 1.146955$$

$$M_4 = \left(\frac{\pi}{8} \right) \left(f \left(\frac{\pi}{16} \right) + f \left(\frac{3\pi}{16} \right) + f \left(\frac{5\pi}{16} \right) + f \left(\frac{7\pi}{16} \right) \right) \\ = \left(\frac{\pi}{8} \right) (0.441690 + 0.745366 + 0.911850 + 0.990346) \\ = 1.213146$$

$$S_8 = \frac{2}{3} M_4 + \frac{1}{3} T_4 = 1.191082$$

$$T_8 = \frac{T_4 + M_4}{2} = 1.180051$$

n	T_n	M_n	S_n
1	0.785398	1.320876	
2	1.053137	1.240773	1.142383
4	1.146955	1.213146	1.178228
8	1.180051		1.191082