Numerical Methods Overview (12/6/21)

Convergence

Q2 [Practice/M]

Complete the following table, where $e_r \approx ke_{r-1}$

$X_{4n} \approx X_{2n} + k(X_{2n} - X_n)$

$X_\infty \approx X_{2n} + \frac{1}{\frac{1}{k}-1}(X_{2n} - X_n)$

(where $X = T, M \text{ or } S$)

	Xth order method	Yth order convergence	k	$\frac{1}{\frac{1}{k}-1}$
Fixed Point	n/a		$g'(\alpha)$	n/a
Newton Raphson	n/a		n/a	n/a
Forward Difference				
Central Difference				
Trapezium rule				
Midpoint rule				
Simpson's rule				

Integration

Q3 [Practice/M]

Obtain the extrapolation formula $T \approx T_{2n} + \frac{1}{3}(T_{2n} - T_n)$

from $T_{2n} - T \approx \frac{T_n - T}{4}$

Q4 [Practice/M]

$\int_0^{\frac{\pi}{2}} \sqrt{sinx}\ dx$ is to be estimated. Complete the following table, by the quickest method. Give values to 6 dp.

n	T_n	M_n	S_n
1			
2			
4			
8			

Q5 [Practice/E]

Use the following Trapezium Rule estimates to obtain extrapolated values for T_{16} and T_∞.

n	T_n
1	0.785398
2	1.053137
4	1.146955
8	1.180051

Q6 [Practice/M]

$\int_0^1 \sqrt{x}\ dx$ is to be estimated. Complete the following table, by the quickest method. Give values to 6 dp.

n	T_n	M_n	S_n
1			
2			
4			
8			

Q7 [Practice/E]

Using the S_n given, complete the following table of ratios of differences (where S is the exact value of $\frac{2}{3}$).

n	S_n	$S_n - S_{\frac{n}{2}}$	Ratios	$S_n - S$	Ratios
1					
2	0.638071				
4	0.656527				
8	0.663079				

Sol'n of Eq'ns

Q1 [Practice/E]

Describe the relative merits of the Secant method and the method of False Position.