

Numerical Methods - Calculator method

(1 page; 22/10/18)

Example: Finding $\sqrt{2}$ by an iterative method

Let (eg) $x_0 = 1.5$ be the first estimate for $\sqrt{2}$

Consider the rectangle with sides x_0 and $\frac{2}{x_0}$

As $x_0 \cdot \frac{2}{x_0} = 2$, one of these sides must be less than $\sqrt{2}$, and the other one greater.

The average of x_0 and $\frac{2}{x_0}$ will then be the next estimate, x_1

$$\text{ie } x_1 = \frac{1}{2} \left(x_0 + \frac{1}{x_0} \right)$$

and so on, with $x_{r+1} = \frac{1}{2} \left(x_r + \frac{2}{x_r} \right)$

On a calculator, try the following (it should work for Casio models):

$$\begin{aligned} 1.5 = \\ 0.5(\text{Ans} + 2 \div \text{Ans}) \\ = [\text{repeatedly}] \end{aligned}$$

You should get the sequence:

1.416666667
1.414215686
1.414213562
1.414213562