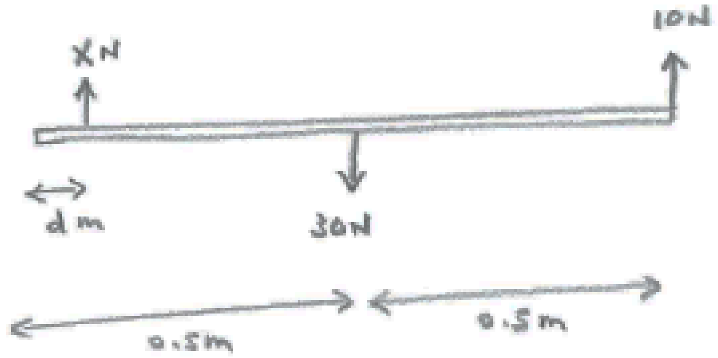
Moments - Exercises (Solutions) (13 pages; 11/3/17)

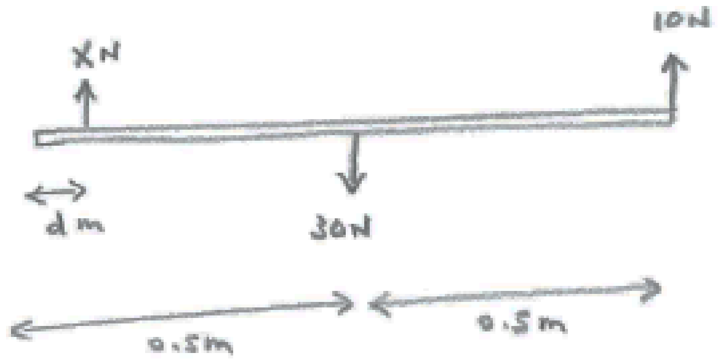
(1) Vertical forces of X, 30 and 10 N are applied to a light rod of length 1 m, as shown in the diagram. The force of X N is applied at a distance of d m from the left-hand end, and the force of 30N is applied at the mid-point of the rod.



(a) What values must X and d have in order for the rod to be in equilbrium?

(b) The force of X N is removed, and the forces of 30N and 10N are to be replaced with a single force having the same effect as these two forces. What is the size and line of action of this single force?

Solution



(a) Vertical equilbrium $\Rightarrow X + 10 = 30 \Rightarrow X = 20$

Taking moments about the right-hand end (for example):

$30(0.5) - 20(1 - d) = 0 \Rightarrow -5 + 20d = 0 \Rightarrow d = 0.25$

[Whenever the forces are balanced, the total moment will be the same about any point; eg taking moments about the mid-point instead:

$10(0.5) - 20(0.5 - d) = 0 \Rightarrow -5 + 20d = 0$, as before

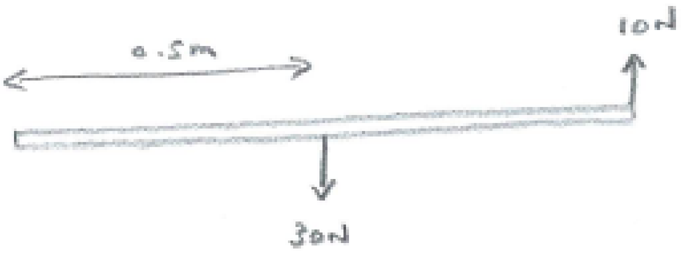
or, about the point where X is applied:

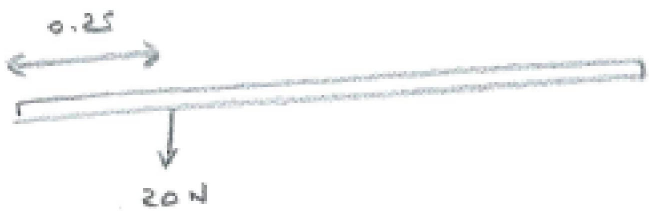
$10(1 - d) - 30(0.5 - d) = 0 \Rightarrow -5 + 20d = 0$]

(b) From (a), X counteracts the effect of the other two forces to give equilibrium.

Now a force of 20 N acting at the same position as X, but in the opposite direction, will also be counteracted by X. Hence it follows that this force is equivalent to the forces of 30 N and 10 N.

Thus the two systems shown below are equivalent.





Alternative method:

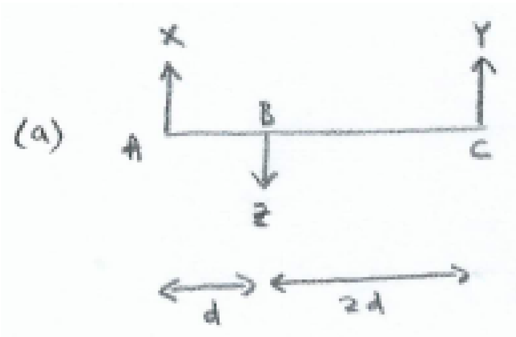
The single equivalent force must be of magnitude 20 N, acting downwards (this being the net effect of the two forces). Suppose that it acts at a distance d from the left-hand end.

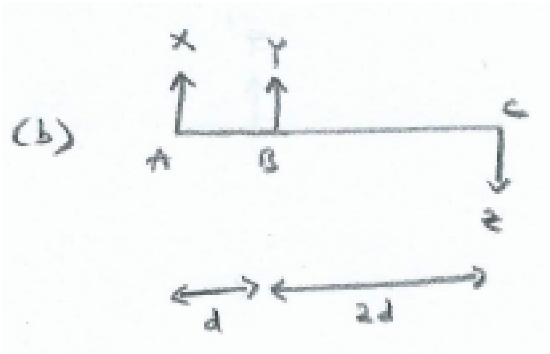
Then we require the moment of this force about the left-hand end (say) to equal the net moment of the two forces.

So $-20d = 10(1) - 30(0.5) = -5 \Rightarrow d = 0.25$

(Note: the single force could not act beyond the left-hand end, as that would give rise to a positive moment, which could not be equated to -5)

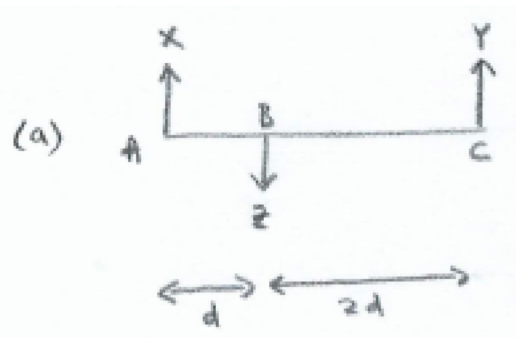
(2) (i) Which of the following systems of forces could be in equilbrium? (with X, Y and Z >0)

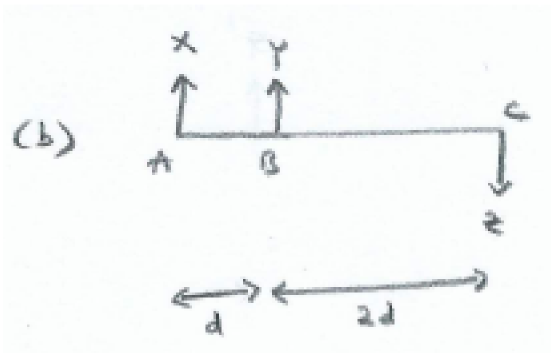




(ii) Assuming that $X + Y = Z$, show that the total moments about A, B and C are equal, in both of the cases in (i).

Solution





(i) (a) Vertical equilibrium requires that $X + Y = Z$.

For rotational equilibrium, taking moments about B, $2dY - dX = 0$, so that $X = 2Y$.

Thus there is equilibrium provided that $Y = \frac{X}{2}$ and $Z = \frac{3X}{2}$.

[Note: As about to be shown in (ii), we can take moments about any point, provided that $X + Y = Z$]

(b) If we take moments about B, we obtain $-dX - 2dZ$, which cannot equal zero. Thus the system cannot be in equilibrium.

[With 3 forces, the directions of the forces must alternate for equilibrium to be possible.]

(ii) (a) $M(A): 3dY - dZ = 3dY - d(X + Y) = d(2Y - X)$

$M(B): 2dY - dX = d(2Y - X)$

$M(C): 2dZ - 3dX = 2d(X + Y) - 3dX = d(2Y - X)$

(b) $M(A): dY - 3dZ = dY - 3d(X + Y) = -d(3X + 2Y)$

$M(B): -dX - 2dZ = -dX - 2d(X + Y) = -d(3X + 2Y)$

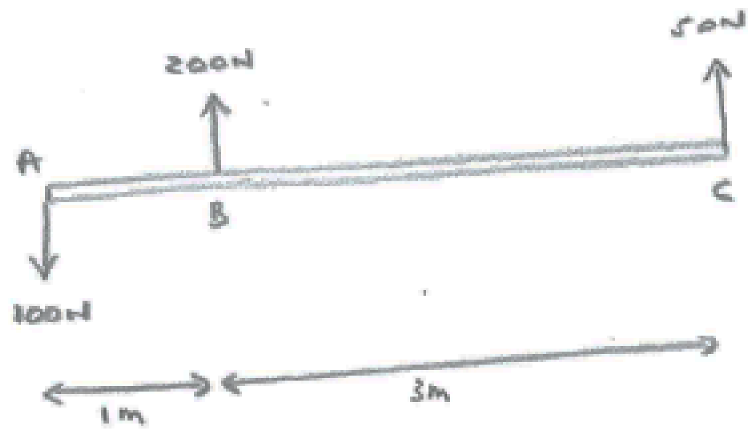
$M(C): -2dY - 3dX = -d(3X + 2Y)$

[Thus the total moment will be the same about any point, provided that the forces balance; regardless of whether there is rotational equilibrium.]

(3) Forces are applied to a light rod, as shown in the diagram.

(a) Find the magnitude and line of action of the additional force that would be needed in order for the rod to be in equilibrium.

(b) Find the magnitude and line of action of the single force that has the same effect as the forces in the diagram.

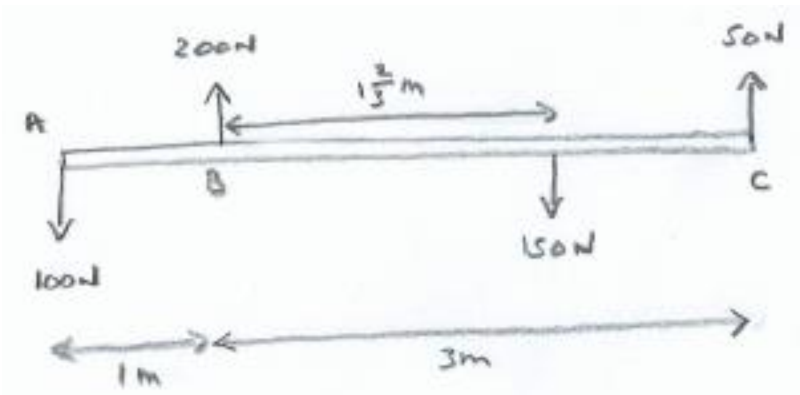


Solution

(a) In order for there to be vertical equilibrium, the additional force must be of magnitude 150N and act downwards. Suppose that its line of action is at a distance d from A.

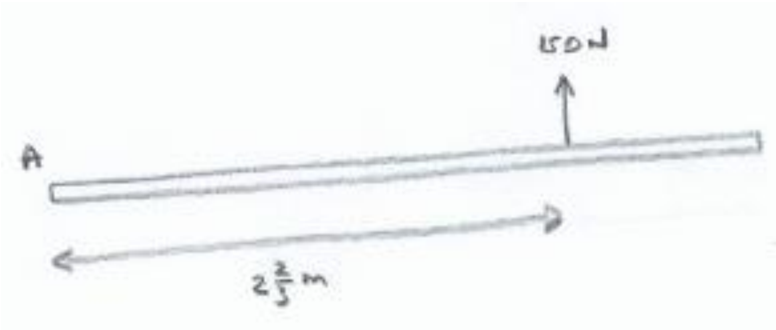
In order for there to be rotational equilibrium, the net moment about A (say) must be zero; ie

$$200(1) + 50(4) - 150d = 0 \Rightarrow d = \frac{400}{150} = \frac{8}{3}\text{ m}$$



(b) **Method 1**

From (a), the force of 150N counteracts the other forces to create equilibrium. This force of 150N also counteracts an equal and opposite force acting at the same point. Hence, this equal and opposite force must have the same effect as the forces in the diagram; ie the required force is as shown below:



Method 2

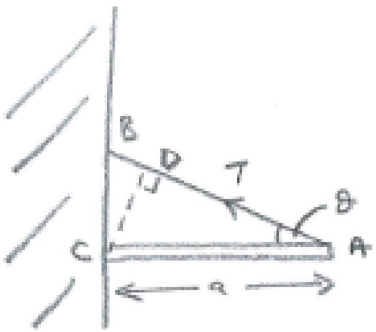
The required force must have the same net value and direction as the forces in the diagram. It therefore has value 150N and acts upwards.

This force will also have the same moment about A (or any other point) as the forces in the diagram.

Suppose that its line of action is at a distance d from A.

Then $150d = 200(1) + 50(4) \Rightarrow d = \frac{400}{150} = \frac{8}{3}$ m

(4)



Show that the moment of T about C is the same:

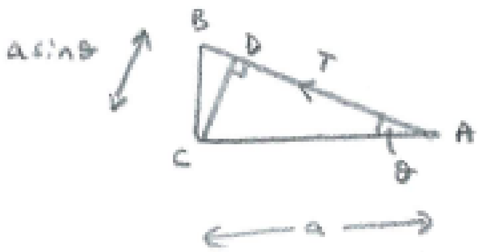
(i) if T is multiplied by CD

(ii) T is resolved into horizontal & vertical components at A

(iii) T is resolved into horizontal & vertical components at B

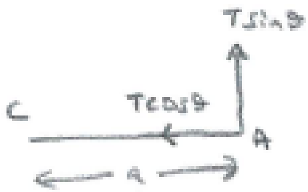
Solution

(i)



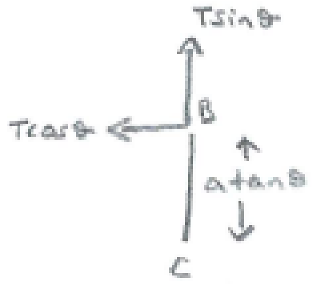
moment $= T \times CD = Tasin\theta$

(ii)



moment $= (Tcos\theta)(0) + (Tsin\theta)a = Tasin\theta$

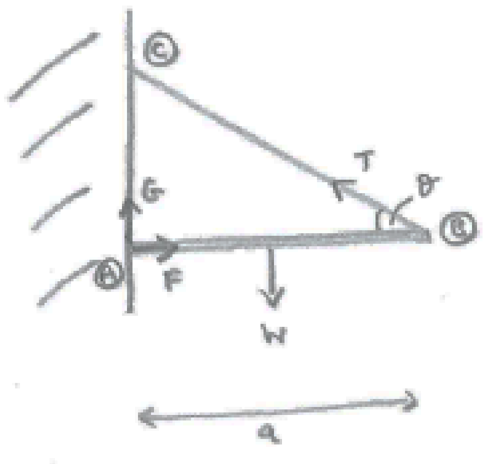
(iii)



Referring to the original diagram, $CB = atan\theta$, so that

moment $= (Tcos\theta)(atan\theta) + (Tsin\theta)(0) = Tasin\theta$

(5) Alternative Moments Methods



A rod AB is attached to a wall at A, and held in a horizontal position by a rope BC.

Show that, as an alternative to resolving forces horizontally and vertically, and taking moments about A, it is also possible to:

(a) resolve forces horizontally and take moments about A & B,

or (b) take moments about A, B & C;

but that it is not possible to do the following:

(c) resolve forces vertically and take moments about A & B,

or (d) take moments about A, B & the midpoint of AB

Solution

Resolving forces horizontally and vertically,

$F = Tcos\theta$ (1) & $W = G + Tsin\theta$ (2)

Taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (3)

Then (3) $\Rightarrow T = \frac{W}{2sin\theta}$

and hence (1) $\Rightarrow F = \frac{Wcot\theta}{2}$

and (2) $\Rightarrow G = W - \frac{W}{2} = \frac{W}{2}$

Following method (a) instead,

resolving horizontally gives $F = Tcos\theta$ (4);

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (5),

and taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (6)

Then from (6), $G = \frac{W}{2}$;

from (5), $T = \frac{W}{2sin\theta}$,

and from (4), $F = \frac{Wcot\theta}{2}$

Following method (b) instead,

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (7);

taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (8),

and taking moments about C gives

$F(atan\theta) - W\left(\frac{a}{2}\right) = 0$ (9)

Then from (8), $G = \frac{W}{2}$;

from (7), $T = \frac{W}{2sin\theta}$,

and from (9), $F = \frac{Wcot\theta}{2}$

However, following method (c):

resolving vertically gives $W = G + Tsin\theta$ (10);

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (11),

and taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (12),

but we have no equation involving F

Also, following method (d):

taking moments about A gives $(Tsin\theta)a - W\left(\frac{a}{2}\right) = 0$ (13);

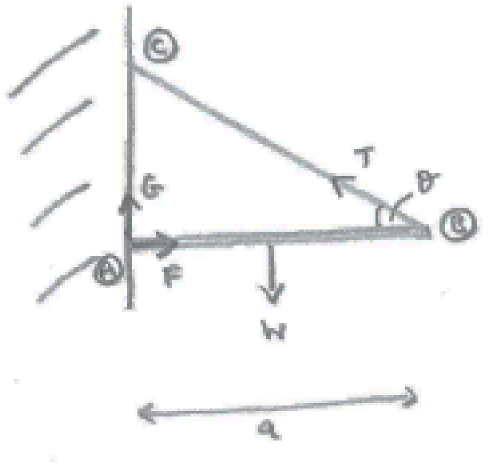
taking moments about B gives $-Ga + W\left(\frac{a}{2}\right) = 0$ (14),

and taking moments about the midpoint of AB gives

$-G\left(\frac{a}{2}\right) + (Tsin\theta)\left(\frac{a}{2}\right) = 0$,

and once again we have no equation involving F

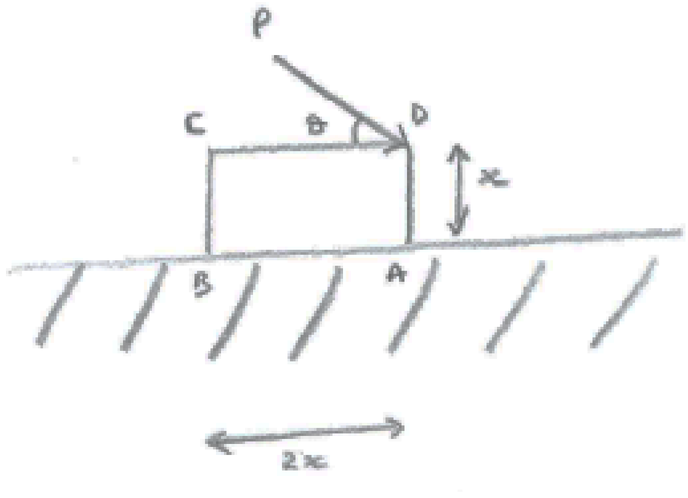
(6) Referring to the diagram below, about which point should moments be taken, in order to find F in terms of W?



Solution

As the lines of action of G & T (the two unwanted forces) pass through C, this is the required point.

(7) A uniform block of mass m rests on a table, and a force P is applied at D, as shown in the diagram. The block has length $2x$ and height x. The coefficient of friction between the block and the table is μ.



(i) If the block is on the point of sliding, find an expression for P.

(ii) If instead the block is on the point of toppling, find an expression for P.

(iii) If the block is to topple before it slides, find a condition on μ.

Solution

(i) The normal reaction, $R = mg + Psin\theta$

The frictional force $= \mu(mg + Psin\theta)$

Hence, at the point of sliding, $\mu(mg + Psin\theta) = Pcos\theta$,

so that $P(cos\theta - \mu sin\theta) = \mu mg$

and $P = \frac{\mu mg}{cos\theta - \mu sin\theta}$

(ii) If the block is on the point of toppling, it will be about A, and the only reaction on the block will be at A. [This will be a combination of a normal reaction and friction.]

As the block is uniform, its weight will act at a distance x from AD, and so, taking moments about A,

$(mg)x = (Pcos\theta)x$

[the normal reaction and friction contribute nothing, as they act at A]

Hence $P = \frac{mg}{cos\theta}$

(iii) At the critical position where the block is about to both slide and topple,

$P = \frac{\mu mg}{cos\theta - \mu sin\theta} = \frac{mg}{cos\theta}$

so that $\mu cos\theta = cos\theta - \mu sin\theta$;

$\mu(cos\theta + sin\theta) = cos\theta$

and $\mu = \frac{cos\theta}{cos\theta + sin\theta} = \frac{1}{1 + tan\theta}$

So, if the block is to topple before it slides, we require

$\mu > \frac{1}{1 + tan\theta}$ [ie making the frictional force greater]

[reasonableness check: if $\theta = 45°$, then $\mu > 0.5$; also, if θ is reduced to 30°, we would expect a higher value of μ to be necessary, in order for toppling to occur first (since the block is now more prone to slide than topple), and the condition gives $\mu > \frac{1}{1 + \frac{1}{\sqrt{3}}} = 0.634$]