Modulus function (3 pages; 4/9/18)

See also: Inequalities, Transformations

(1) **Example**: $y = |x - 2| + 1$

Approach A: Case by case

Case (i) : $x - 2 \geq 0$

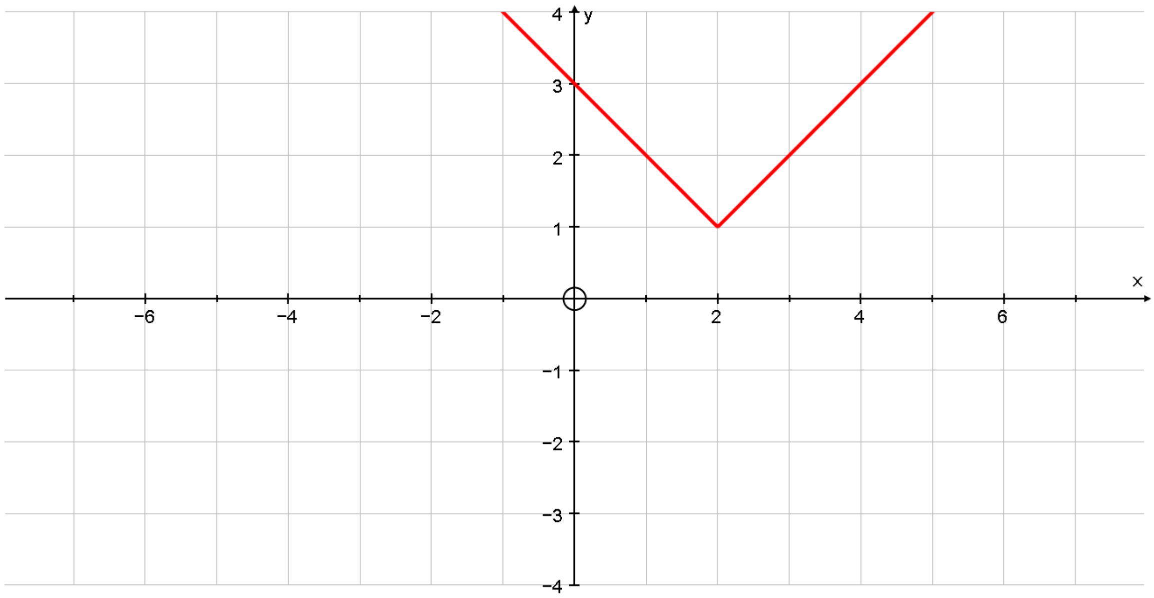
$\Rightarrow y = x - 2 + 1 = x - 1$

Case (ii) : $x - 2 < 0$

$\Rightarrow y = (2 - x) + 1 = 3 - x$

So, for $x < 2$ we have the straight line $y = 3 - x$, and for $x \geq 2$ we have the straight line $y = x - 1$

In order to draw the graph, it makes sense to plot the point (2 , 1), which is common to both lines, and then to draw the two lines by noting their gradients (as below).



Approach B: Analogy with y = $(x-2)^2+1$

The function $y = |x - 2|$ is similar to $y = (x-2)^2$, in that they both take non-negative values only and are symmetrical about $x = 2$.

The quadratic function $y = (x-2)^2 + 1$ has a minimum at (2 , 1) [since

$(x-2)^2$ has its smallest value (of 0) when $x - 2 = 0$]

and the same is true for $y = |x-2| + 1$

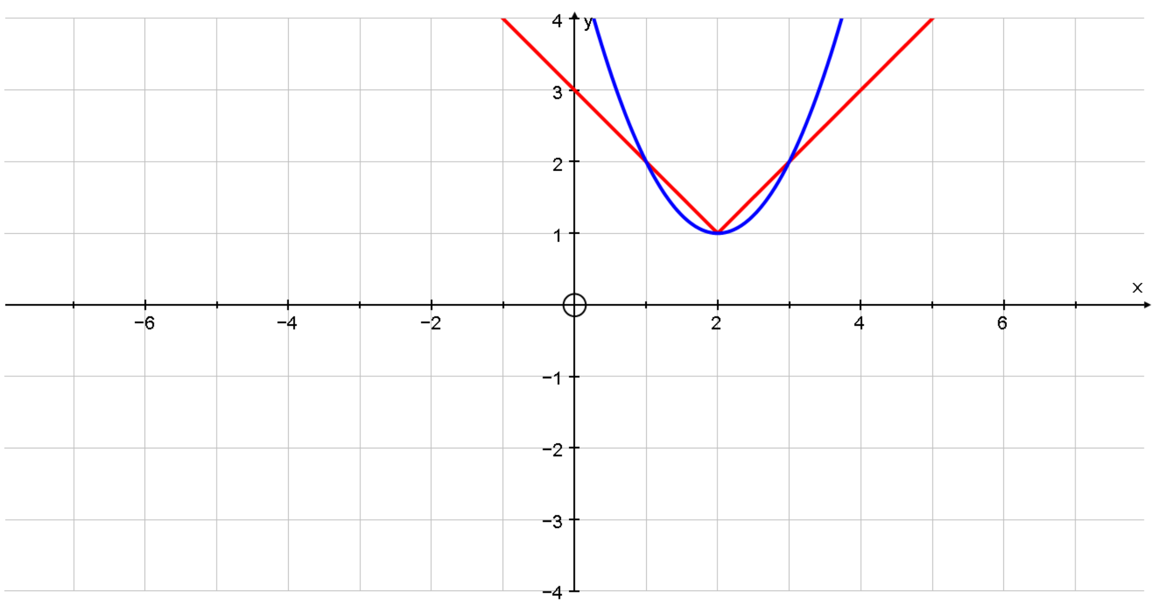
Then the shape of the graph will be similar to that of

$y = (x-2)^2 + 1$, except of course that it is straight rather than curved.

The right hand branch is simply the line $y = (x-2) + 1 = x - 1$,

which (once again) is most easily drawn by noting its gradient.

The left hand branch can then be obtained by symmetry.



(2) eg for $y = |x - 1| + |x + 2|$,

consider the cases $x \leq -2, -2 < x < 1, x \geq 1$

(3) For $y = |f(x)|$, when $f(x) = 0$, there will be a cusp.

Note when sketching the curve that $f'(x_0 + \delta) = -f'(x_0 - \delta)$, but that the shape of the curve may differ significantly away from x_0.