Modulus Function (2 pages; 2/6/23)

(1) The modulus sign is difficult to manipulate mathematically. However, the modulus function can be broken down, as follows:

When $x \geq 0, |x| = x$

When $x < 0, |x| = -x$

(2) Note that $\sqrt{x^2} = |x|$ (as, by convention, the square root symbol denotes the positive root [consider the quadratic formula, which needs a $\pm\sqrt{\ }$ for this reason]).

(3) Example: $y = |x - 2| + 1$

Case (i) : $x - 2 \geq 0$

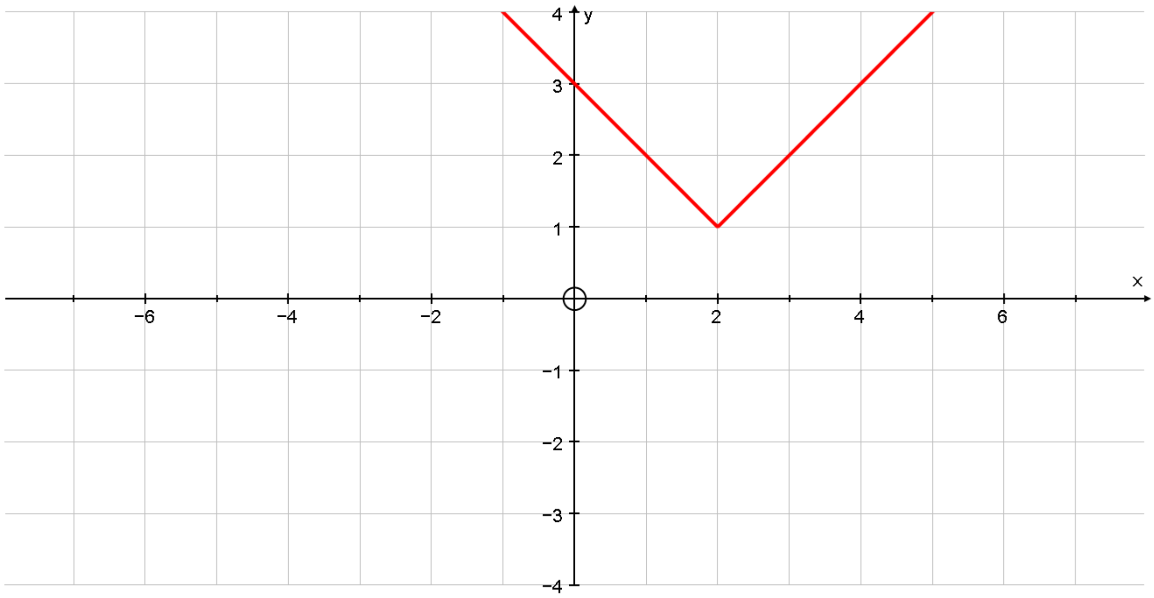
$\Rightarrow y = x - 2 + 1 = x - 1$

Case (ii) : $x - 2 < 0$

$\Rightarrow y = -(x - 2) + 1 = 3 - x$

So, for $x < 2$ we have the straight line $y = 3 - x$, and for $x \geq 2$

we have the straight line $y = x - 1$.



[Analogy with $y = (x-2)^2 + 1$: The function : $y = |x-2|$ is similar to $y = (x-2)^2$, in that they both take non-negative values only and are symmetric about $x = 2$.

The quadratic function $y = (x-2)^2 + 1$ has a minimum at (2 , 1)

and the same is true of $y = |x-2| + 1$.]

(4) For $y = |x-1| + |x+2|$, consider the cases

$x \leq -2, -2 < x < 1, x \geq 1$

(5) For $y = |f(x)|$, when $f(x) = 0$, there will be a cusp.

Note when sketching the curve that $f'(x_0 + \delta) = -f'(x_0 - \delta)$.