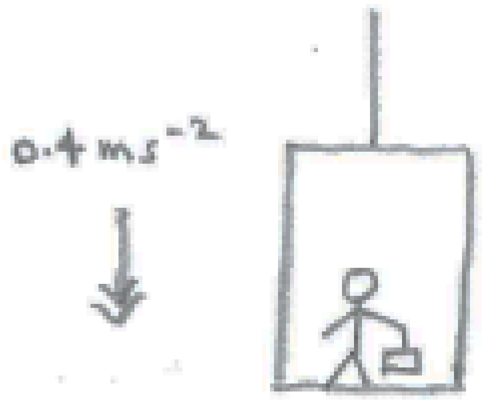
Mechanics Exercises - Misc (Solutions)

(5 pages; 26/1/19)

(1) Forces

A man is in a lift, which is moving downwards with an acceleration of $0.4ms^{-1}$. The lift is suspended by a cable, and the man is holding a parcel by a light string, as in the diagram. The masses of the lift, man and parcel are 300kg, 80kg and 5kg, respectively.



(i) Find :

(a) the tension in the cable

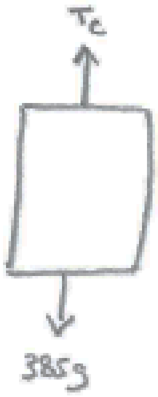
(b) the reaction between the man and the floor of the lift

(c) the tension in the string

(ii) Does the man feel heavier or lighter than he would if the lift were stationary and he were no longer carrying the parcel?

Solution

(a) Considering the lift, man and parcel as a single object:



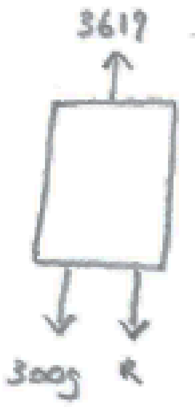
[T_C and 385g are the external forces]

$N2L \Rightarrow 385g - T_C = 385(0.4)$, where T_C is the tension in the cable

[any new symbols introduced need to be defined in an exam answer]

$\Rightarrow T_C = 385(9.8 - 0.4) = 3619\ N$

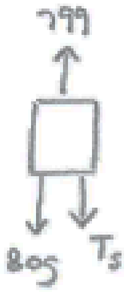
(b) Considering the forces on the lift:



$N2L \Rightarrow 300g + R - 3619 = 300(0.4)$, where R is the reaction between the man and the floor

$\Rightarrow R = 3619 + 120 - 300(9.8) = 799N$

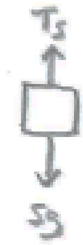
(c) Considering the forces on the man:



$N2L \Rightarrow 80g + T_S - 799 = 80(0.4)$, where T_S is the tension in the string

$\Rightarrow T_S = 799 + 32 - 80(9.8) = 47N$

[Check: Considering the forces on the parcel:



$N2L \Rightarrow 5g - T_S = 5(0.4)$

$\Rightarrow T_S = 5(9.8) - 2 = 47N$]

(ii) If the lift is stationary and the man is not carrying the parcel, the reaction between himself and the floor is just his weight [see note below]: $80(9.8) = 784N$

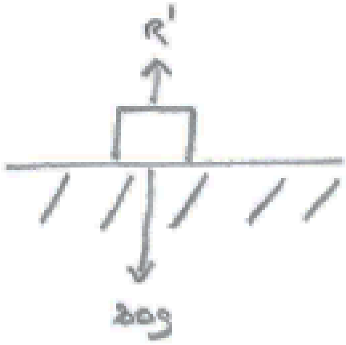
Thus he feels heavier, as $799 > 784$.

[The apparent gravity is now $9.8 - 0.4 = 9.4$, but the man's weight has effectively been increased by 5kg, giving a net apparent weight of

$85 \times 9.4 = 799N$ (this is a check on (b))]

Note: In the stationary situation (with no parcel),

$N2L \Rightarrow 80g - R' = 0 \Rightarrow R' = 80g$; ie the man's weight



(2) Friction

A sledge with a child onboard is being pulled along on level ground, at a constant speed, by means of a rope inclined at 30° to the horizontal. The sledge and child together have a mass of $100kg$. The coefficient of friction between the sledge and the ground is $\frac{1}{10}$. Assuming that $g = 10$, find the tension in the rope.

Solution

Let T be the tension, and let R be the normal reaction of the ground on the sledge. Then, applying N2L vertically:

$R + Tsin30° = 100g$

Applying N2L horizontally, $Tcos30° = \mu R$

Hence $T\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{10}\left(1000 - \frac{T}{2}\right)$,

so that $T\left(\frac{\sqrt{3}}{2} + \frac{1}{20}\right) = 100$

and $T = 109\ N\ (3sf)$

(3) Energy

A car of mass 1 tonne starts to climb a hill at $20ms^{-1}$. The slope of the hill is a constant θ, where $sin\theta = \frac{1}{10}$. If the car is not accelerating (or braking) and there is a constant resistance to motion of $1000N$, find the speed of the car when it has gained a height of $5m$. Assume that $g = 10$.

Solution

Method 1

By the Work-Energy principle,

Gain in KE = Work done by forces,

so that $\frac{1}{2}(1000)(v^2 - 20^2) = -1000g(5) - 1000\left(\frac{5}{sin\theta}\right)$

$\Rightarrow 500v^2 = 200000 - 50000 - 50000$

$\Rightarrow v^2 = 200 \Rightarrow v = 14.1\ ms^{-1}\ (3sf)$

Method 2

By Conservation of Energy,

Gain in PE = loss of KE − work done against resistance

$\Rightarrow 1000g(5) = \frac{1}{2}(1000)(20^2 - v^2) - 1000\left(\frac{5}{sin\theta}\right)$

which gives the same equation.