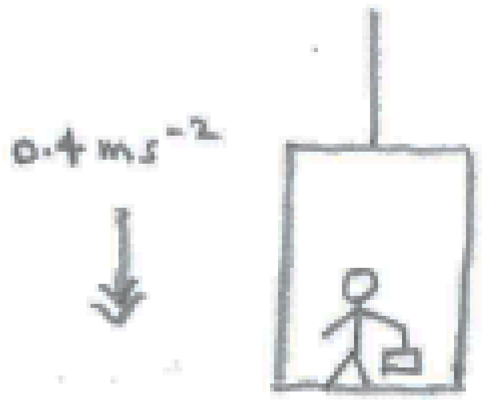
# Mechanics Exercises - Misc (Solutions)

(16 pages; 8/3/19)

## (1) Forces

A man is in a lift, which is moving downwards with an acceleration of $0.4ms^{-1}$. The lift is suspended by a cable, and the man is holding a parcel by a light string, as in the diagram. The masses of the lift, man and parcel are 300kg, 80kg and 5kg, respectively.




(i) Find :

(a) the tension in the cable

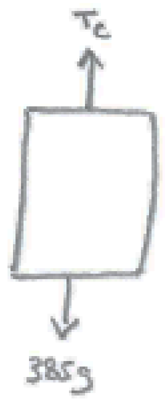
(b) the reaction between the man and the floor of the lift

(c) the tension in the string

(ii) Does the man feel heavier or lighter than he would if the lift were stationary and he were no longer carrying the parcel?

## Solution

(a) Considering the lift, man and parcel as a single object:



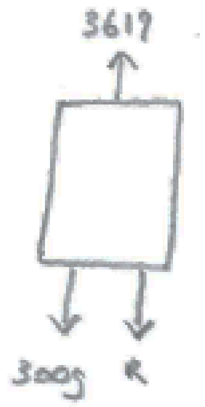

[$T_C$ and 385g  are the external forces]

$N2L \Rightarrow 385g - T_C = 385(0.4)$ , where $T_C$ is the tension in the cable

[any new symbols introduced need to be defined in an exam answer]

$\Rightarrow T_C = 385(9.8 - 0.4) = 3619\ N$

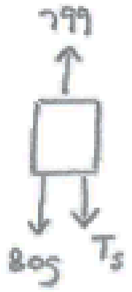
(b) Considering the forces on the lift:




$N2L \Rightarrow 300g + R - 3619 = 300(0.4)$ , where R is the reaction between the man and the floor

$\Rightarrow R = 3619 + 120 - 300(9.8) = 799N$

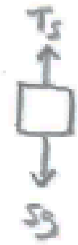
(c) Considering the forces on the man:




$N2L \Rightarrow 80g + T_S - 799 = 80(0.4)$ , where $T_S$ is the tension in the string

$\Rightarrow T_S = 799 + 32 - 80(9.8) = 47N$

[Check: Considering the forces on the parcel:




$N2L \Rightarrow 5g - T_S = 5(0.4)$

$\Rightarrow T_S = 5(9.8) - 2 = 47N$ ]

(ii) If the lift is stationary and the man is not carrying the parcel, the reaction between himself and the floor is just his weight [see note below]: $80(9.8) = 784N$

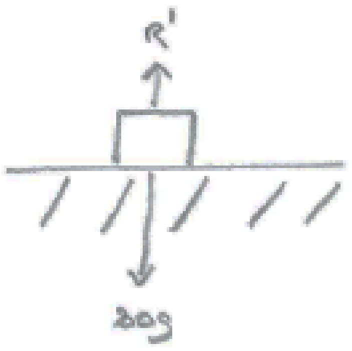
Thus he feels heavier, as $799 > 784$.

[The apparent gravity is now $9.8 - 0.4 = 9.4$, but the man's weight has effectively been increased by 5kg, giving a net apparent weight of

$85 \times 9.4 = 799N$ (this is a check on (b)) ]

**Note**: In the stationary situation (with no parcel),

$N2L \Rightarrow 80g - R' = 0 \Rightarrow R' = 80g$ ; ie the man's weight




## (2) Friction

A sledge with a child onboard is being pulled along on level ground, at a constant speed, by means of a rope inclined at 30° to the horizontal. The sledge and child together have a mass of $100kg$. The coefficient of friction between the sledge and the ground is $\frac{1}{10}$. Assuming that $g = 10$, find the tension in the rope.

### Solution

Let $T$ be the tension, and let $R$ be the normal reaction of the ground on the sledge. Then, applying N2L vertically:

$R + Tsin30° = 100g$

Applying N2L horizontally, $Tcos30° = \mu R$

Hence $T\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{10}\left(1000 - \frac{T}{2}\right)$,

so that $T\left(\frac{\sqrt{3}}{2} + \frac{1}{20}\right) = 100$

and $T = 109\ N\ (3sf)$

## (3) Energy

A car of mass 1 tonne starts to climb a hill at $20ms^{-1}$. The slope of the hill is a constant $\theta$, where $sin\theta = \frac{1}{10}$. If the car is not accelerating (or braking) and there is a constant resistance to motion of $1000N$, find the speed of the car when it has gained a height of $5m$. Assume that $g = 10$.

**Solution**

**Method 1**

By the Work-Energy principle,

Gain in KE = Work done by forces,

so that $\frac{1}{2}(1000)(v^2 - 20^2) = -1000g(5) - 1000\left(\frac{5}{sin\theta}\right)$

$\Rightarrow 500v^2 = 200000 - 50000 - 50000$

$\Rightarrow v^2 = 200 \Rightarrow v = 14.1\ ms^{-1}\ (3sf)$

**Method 2**

By Conservation of Energy,

Gain in PE = loss of KE − work done against resistance

$\Rightarrow 1000g(5) = \frac{1}{2}(1000)(20^2 - v^2) - 1000\left(\frac{5}{sin\theta}\right)$

which gives the same equation.

## (4) Oscillations

A lift has an elastic string suspended from its ceiling, with a mass of 10 grams at the end of the string. The string has natural length 80 $cm$, and modulus of elasticity $20N$. Initially, when the lift is stationary, the mass is hanging in equilibrium. The lift then starts to ascend with an acceleration of 0.2 $ms^{-2}$. Show that the extension of the string after $t$ $secs$ is $0.4 - 0.008\cos(50t)$ $cm$.

[Assume that $g = 9.8ms^{-2}$]

**Solution**

Let $x$ be the distance of the mass below the level of the ceiling of the lift when it is stationary, measured relative to the lift's surroundings.

Then $x = 0.8 + e - y$,

where $e$ is the extension of the string and $y$ is the distance moved (upwards) by the lift,

so that $\ddot{x} = \ddot{e} - \ddot{y} = \ddot{e} - 0.2$

Considering the forces on the mass,

$0.01g - T = 0.01\ddot{x}$ ,

where $T$ is the tension in the string,

and by Hooke's law, $T = \frac{20}{0.8}e$

So $0.01g - \frac{20}{0.8}e = 0.01\ddot{x} = 0.01(\ddot{e} - 0.2)$,

and hence $g - 2500e = \ddot{e} - 0.2$,

or $\ddot{e} + 2500e = 9.8 + 0.2 = 10$ (*)

To solve the differential equation:

the auxiliary equation is $\lambda^2 + 2500 = 0$, with roots $\lambda = \pm 50i$,

so that the complementary function is $Ae^{50it} + Be^{-50it}$

or $(A + B)cos50t + (A - B)isin50t$

or $Ccos50t + Dsin50t$,

which can be written as $Ecos(50t + \alpha)$

The particular integral of the differential equation is a constant F

(as the RHS of (*) is a constant),

such that $2500F = 10$, so that $F = 0.004$

Thus the general solution of (*) is:

$e = Ecos(50t + \alpha) + 0.004$ (**)

and $\dot{e} = -50Esin(50t + \alpha)$

When $t = 0$, and the mass is hanging in equilibrium,

$0.01g - T = 0$ and $T = \frac{20}{0.8}e$,

so that $0.01g = \frac{20}{0.8}e$ and $e = \frac{49}{12500}$

Also, at $t = 0$, $\dot{e} = 0$, so that $\alpha = 0$

Thus, from (**), $\frac{49}{12500} = E + 0.004$,

and $E = -0.00008$,

so that $e = 0.004 - 0.00008cos(50t)\ m$

or $0.4 - 0.008cos(50t)\ cm$

## (5) Oscillations

A flexible bar is embedded horizontally in a wall. A particle rests on the free end of the bar, and the bar (with the particle) is pulled down 2cm below the horizontal, and then released. Given that the bar and particle start to perform simple harmonic motion about the horizontal position, with 5 cycles per second, how long is it before the particle loses contact with the bar, and what speed does it have at that point? [Note: The particle will not be in contact with the bar long enough to complete a cycle of the simple harmonic motion.]

### Solution

Let $x$ be the displacement of the bar from the horizontal, where upwards is the positive direction.

Then (converting to S.I. units), $x = 0.02sin(\omega t)$, if (for convenience) we measure time from when $x = 0$.

Then, as 1 cycle (ie $2\pi$ radians) takes $\frac{1}{5}$ sec.,

$\omega\left(\frac{1}{5}\right) = 2\pi$, so that $\omega = 10\pi$

and $\ddot{x} = -\omega^2 x$

The particle is subject to gravity and a reaction force $R(x)$ from the bar, so that

$R(x) - mg = m\ddot{x}$ , whilst the particle is in contact with the bar

(where $m$ is the mass of the particle)

The particle loses contact with the bar when $R(x) = 0$;

ie when $-mg = m\ddot{x}$ and $\ddot{x} = -g$

As $\ddot{x} = -\omega^2 x$, this occurs when $-\omega^2 x = -g$; ie $x = \frac{g}{\omega^2}$

(note that this is where $x > 0$; ie above the horizontal).

The bar and particle take a quarter of a cycle, ie $\frac{1}{20}$ sec., to travel from the point of release to the horizontal.

If contact is lost at $T$ secs after the horizontal has been reached,

$\frac{g}{\omega^2} = x = 0.02sin(\omega T)$ , where $\omega = 10\pi$.

Then $T = \frac{1}{10\pi} sin^{-1}\left(\frac{9.8}{100\pi^2(0.02)}\right) = 0.016537$

and hence the required time from release is $\frac{1}{20} + 0.016537 = 0.066537 = 0.0665$ s (3sf)

The speed of the particle is $\dot{x}(T) = 0.02(10\pi)\cos(10\pi T)$

$= 0.54542\ ms^{-1}$ or $54.5cms^{-1}$(3sf)

## (6) Equilibrium

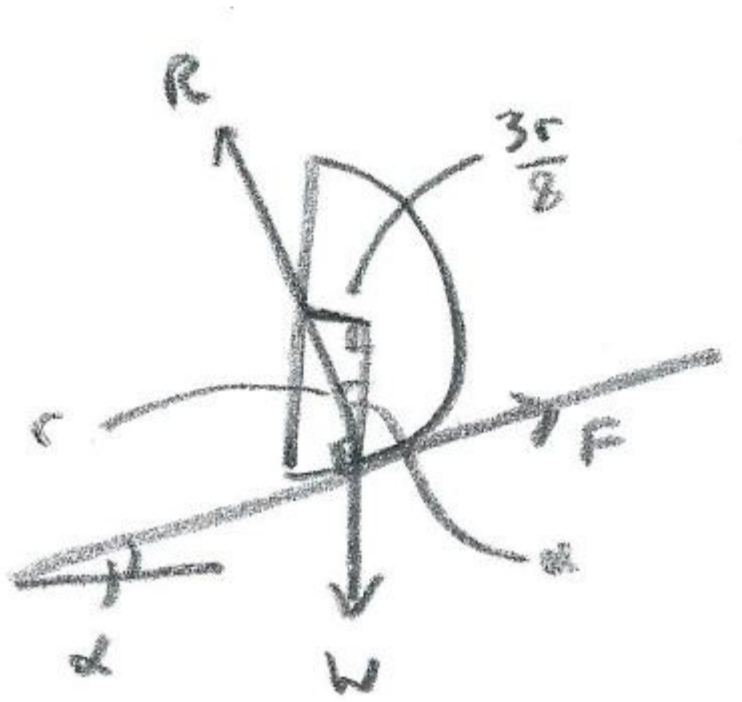
A uniform solid hemisphere rests in equilibrium on a rough slope, with its curved surface in contact with the slope, which is inclined at an angle $\alpha$ to the horizontal, in such a way that the plane face of the hemisphere is vertical. Find $\alpha$.

## Solution

In order to establish the necessary configuration of the hemisphere and slope, we note that the weight of the hemisphere must act on a line that passes through the point of contact between the hemisphere and the slope. [The three forces acting on the hemisphere (its weight, the reaction from the slope and friction) will then all meet at a single point, as is required for a body in equilibrium that is subject to three forces - otherwise a non-zero moment would exist about the point of intersection of two of the forces).

A diagram can be drawn by starting with the hemisphere, and adding in the slope. Note also that the point of contact will be on a tangent to the hemisphere, and that the perpendicular to the tangent, along which the reaction force acts, will be a radius of the hemisphere.

The radius of the hemisphere (which we are expecting to cancel out, as it isn't mentioned in the question) can be taken to be $r$.




The weight can be taken to act at the centre of mass of the hemisphere, which is at a distance $\frac{3r}{8}$ from the plane face.

From the diagram, $sin\alpha = \frac{\left(\frac{3r}{8}\right)}{r} = \frac{3}{8}$, and hence $\alpha = 22.0°$ (1dp).

## (7) Equilibrium

A stepladder is made up of two sides, which have weights 80N and 8N. Both sides are of length 2m. There is a platform resting on the top, which together with a person standing on it weighs 700N. The two sides are also joined together by a horizontal light rope of length 1m, which starts at a distance of 0.6m along each side, from the base. See Fig. 1. There is no friction between the ladder and the ground, or between the platform and the ladder. Find the tension in the rope.

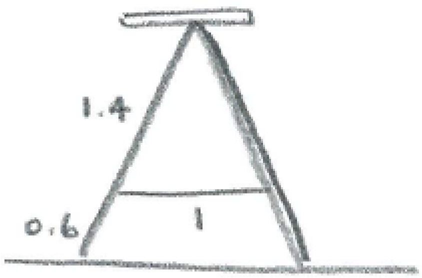


Fig. 1

### Solution

[Equations can be obtained by applying N2L and/or taking moments for either individual components (eg one side of the stepladder), or the whole system. There will be a limit to the number of independent equations that can be created. But because some equations may prove to be redundant anyway (as we will see), it may not be worth attempting to ensure that all the equations are independent. Instead, we can just create any equations that look useful, and run the risk of some duplication. Not all of the equations created below are needed for the purpose of finding T, but may be of use for establishing other forces - or combinations of forces.]

Fig. 1a shows the various lengths involved.

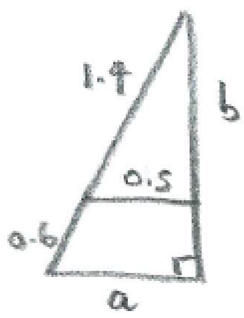



Fig. 1a

Considering the external forces on the ladder (including the rope, but excluding the platform) (Fig. 2),

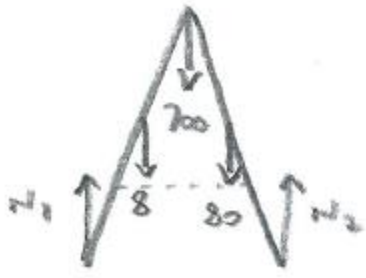



Fig. 2

$N_1 + N_2 = 788$ (1)

Also, taking moments about the base of the righthand side of the ladder :

$-N_1(2a) + (8)\left(\frac{3a}{2}\right) + 700a + 80\left(\frac{a}{2}\right) = 0$ (2),

so that $N_1 = \frac{1}{2}(12 + 700 + 40) = 376$, and hence $N_2 = 412$

Fig. 3 shows the force diagram for the lefthand side of the ladder (excluding the rope and the platform), where $X$ & $Y$ are the components of the reaction from the righthand side of the ladder, and $R_1$ is the reaction from the platform and person.

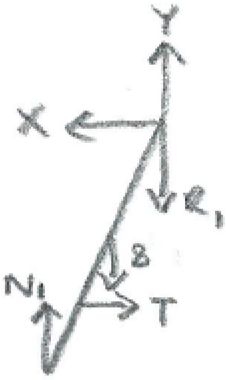



Fig.3

Horizontally this gives $T = X$ (3)

and vertically: $N_1 + Y = 8 + R_1$,

As $N_1 = 376,\ 368 + Y = R_1$ (4)

By N3L, the reaction forces on the righthand side of the ladder from the lefthand side are $X$ & $Y$, as shown in Fig.4. And $R_2$ is the reaction from the platform and person.

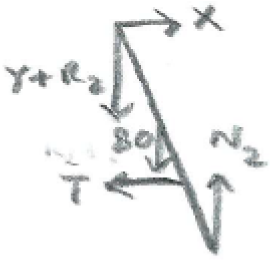



Fig. 4

Then, vertically: $N_2 = 80 + Y + R_2$

As $N_2 = 412,\ 332 = Y + R_2$ (5)

(Horizontally just reproduces $T = X$)

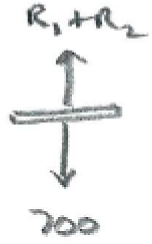



Fig.5

Fig. 5 shows the forces on the platform and person, giving

$R_1 + R_2 = 700$  (6)

[Note that this could have been obtained by adding (4) & (5).]

In order to involve the location of T, we can take moments about the top of the ladder, for say the lefthand side, to give:

$-N_1 a + Tb + (8)\left(\frac{a}{2}\right) = 0$  (6),

so that $T = \frac{a}{b}(376 - 4) = \frac{372a}{b}$

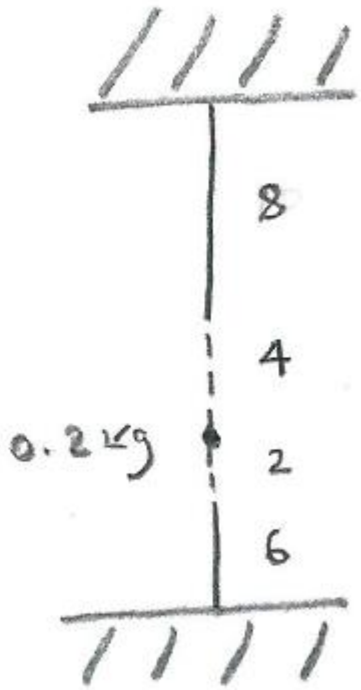
Also, from Fig. 1a, by similar triangles, $\frac{a}{0.5} = \frac{2}{1.4}$, so that $a = \frac{5}{7}$,

and $b^2 = 1.4^2 - 0.5^2$, so that $b = \frac{\sqrt{171}}{10} = \frac{3\sqrt{19}}{10}$,

and hence $T = \frac{372(5)(10)}{7(3\sqrt{19})} = 203.197$

## (8) Oscillations

A 0.2 kg mass is held between two elastic strings, as shown in the diagram. The upper string has original length 8m and modulus of elasticity 2N, and is initially extended by 4m. The lower string has original length 6m and modulus of elasticity 1N, and is initially extended by 2m. When the mass is released, determine its subsequent motion (assume $g = 10$, and ignore any resistance to motion).




### Solution

Let $x$ be the displacement of the mass above its initial position.

By N2L, $(0.2)\ddot{x} = \frac{4-x}{8}(2) - \frac{2+x}{6}(1) - (0.2)(10)$

so that $\ddot{x} = 5\left(1 - \frac{1}{3} - 2\right) - 5x\left(\frac{1}{4} + \frac{1}{6}\right) = -\frac{20}{3} - \frac{25x}{12}$

### Method 1

Writing $\ddot{x} + \frac{25x}{12} = -\frac{20}{3}$ gives a complementary function of

$x = Acos(\frac{5}{\sqrt{12}}t + \alpha)$

and a trial function of $x = C$ for the particular integral gives

$\frac{25C}{12} = -\frac{20}{3}$, so that $C = -\frac{16}{5}$

and the general solution is $x = Acos\left(\frac{5}{\sqrt{12}}t + \alpha\right) - \frac{16}{5}$

so that $\dot{x} = -\frac{5}{\sqrt{12}}Asin\left(\frac{5}{\sqrt{12}}t + \alpha\right)$

Then $t = 0, \dot{x} = 0 \Rightarrow \alpha = 0$

and $t = 0, x = 0 \Rightarrow 0 = A - \frac{16}{5}$

so that the particular solution is $x = \frac{16}{5}[cos\left(\frac{5}{\sqrt{12}}t\right) - 1]$

This is SHM of amplitude $\frac{16}{5}$ about $x = -\frac{16}{5}$

The period of oscillations is given by $\frac{5}{\sqrt{12}}T = 2\pi$,

and is $T = \frac{2\pi\sqrt{12}}{5} = 4.35$ s (3sf)

Thus the mass falls when released, and rises again to its initial position (as would be predicted by conservation of energy).

**Method 2**

$\ddot{x} = -\frac{20}{3} - \frac{25x}{12}$

Let $-\frac{20}{3} - \frac{25x}{12} = -\frac{25y}{12}$, so that $y = x + \frac{20}{3}\left(\frac{12}{25}\right) = x + \frac{16}{5}$

Then $\ddot{y} = \ddot{x} = -\frac{25y}{12}$

which gives SHM of amplitude $\frac{16}{5}$ about $y = 0$; ie $x = -\frac{16}{5}$