

Matrices - Exercises: General - Advanced

(1 page; 30/10/18)

(1) Find the condition(s) for two 2×2 matrices to commute.

(2) Given that a 3×3 determinant can always be reduced to triangular form (in the same way as simultaneous equations), to

produce a multiple of $\begin{vmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{vmatrix}$, show that it can be further

reduced to a multiple of the Identity matrix. [Obviously this is an academic exercise, as the determinant can be evaluated as soon as triangular form has been reached.]

(3) Show that a matrix is orthogonal if and only if

(i) its columns are mutually orthogonal (ie perpendicular, so that their scalar product is zero), and

(ii) each column has unit magnitude

(4) Find c, a & b such that $\begin{pmatrix} 2 \\ 3 \\ c \end{pmatrix} = a \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + b \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$

[ie such that the 3 vectors are not linearly independent]