Matrices – Q6: Eigenvectors [Problem/H](2/6/21)

Given that the eigenvalues of $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ are 4, 4 and 1, establish the geometrical significance of the eigenvectors.

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Solution

First of all, the eigenvector associated with the eigenvalue of 1 will be a line of invariant points (through the Origin) [All eigenvectors are invariant lines through the Origin, and are lines of invariant points when the eigenvalue is 1.]

[When there are repeated eigenvalues, there will either be an invariant plane or an invariant line. When there aren't repeated eigenvalues, there can only be an invariant line. See "Matrices - notes".]

$$\begin{pmatrix} 3-4 & -1 & 1 \\ -1 & 3-4 & 1 \\ 1 & 1 & 3-4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

giving $-x - y + z = 0$ (3 times)

This is the equation of a plane; ie the invariant plane of the transformation (all points map to another point in the plane.)