

Matrices – Shears

Q54 [Practice/M] (6/3/23)

Show that $\frac{(a-1)^2+c^2}{c} = -\frac{b^2+(1-d)^2}{b}$ for the shear $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$.

[It can be assumed that the trace of a 2×2 matrix will equal 2 in the case of a shear.]

Solution

$$a + d = 2 \text{ and } ad - bc = 1$$

The required result is equivalent to

$$b(a - 1)^2 + bc^2 + cb^2 + c(1 - d)^2 = 0 \quad (1)$$

$$\text{As } 1 - d = 1 - (2 - a) = a - 1,$$

$$(1) \Leftrightarrow (a - 1)^2(b + c) + bc(b + c) = 0$$

$$\Leftrightarrow [(a - 1)^2 + bc](b + c) = 0$$

$$\text{And } [(a - 1)^2 + bc = (a - 1)(1 - d) + bc$$

$$= -(ad - bc) + (a + d) - 1$$

$$= -1 + 2 - 1 = 0 \text{ as required.}$$