

Matrices – Q5: Eigenvectors [Problem/M](2/6/21)

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Solution

Let $M\underline{x} = \lambda\underline{x}$ and $N\underline{x} = \mu\underline{x}$

Then $(MN)\underline{x} = M(N\underline{x}) = M(\mu\underline{x}) = \mu(M\underline{x}) = \mu\lambda\underline{x}$

Thus MN , and similarly NM , also have this same eigenvector, and the associated eigenvalue is the product of the corresponding eigenvalues for M & N .