Matrices – Q48: Eigenvectors [Practice/M] (8/6/21)

Consider the transformation represented by $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

(i) What type of transformation is this?

(ii) Use eigenvectors to find the invariant lines through the Origin.

(iii) What can be said about the line $y = 0$?

(iv) What can be said about the line $y = c$? , where $c \neq 0$

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Solution

(i) A shear in the x-direction [as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is unchanged and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ moves parallel to the x-axis]; the image of a point not on the x-axis would also need to be specified, in order to define the shear fully.

(ii)&(iii) $\begin{vmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)^2 = 0 \Rightarrow \lambda = 1$

To find the associated eigenvector: $\begin{pmatrix} 1-1 & 2 \\ 0 & 1-1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$,

so that $2y = 0$; ie the eigenvector is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which means that $y = 0$ is an invariant line, and also a line of invariant points (as $\lambda = 1$)

(iv) As the transformation is a shear, lines parallel to the line of invariant points will be invariant lines.

Note: General invariant lines can be obtained by finding solutions to $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} x' \\ mx'+c \end{pmatrix}$