Matrices – Q41: Transformations [Problem/M] (4/6/21)

Derive a formula for the area of a triangle with corners at $(0,0), (a, b), (c, d)$, using matrix transformations.

Derive a formula for the area of a triangle with corners at $(0,0), (a, b), (c, d)$, using matrix transformations.

Solution

The formula for the area of a triangle with corners $(0,0), (a, b), (c, d)$ can be obtained by considering the matrix transformation $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$: (a, b) is the image of $(1,0)$ and (c, d) is the image of $(0,1)$; the area of the triangle with corners $(0,0), (1,0), (0,1)$ is $\frac{1}{2}$, and the area scale factor is $|ad - bc|$, since $ad - bc$ is the determinant of the matrix (the modulus sign only being needed when the order of the corners becomes reversed in the course of the transformation). So the required area is $\frac{1}{2}|ad - bc|$.