Matrices – Q40: Transformations [Problem/M] (4/6/21)

(i) Find the equation of the line that the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ maps all points to.

(ii) For the same transformation, find the equation of the line that maps to the point with an x-coordinate of w.

(iii) For the same transformation, for which point(s) will the x-coordinate remain unchanged by the transformation?

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Solution

(i) $\begin{pmatrix}1 & 2\\3 & 6\end{pmatrix}\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}u\\v\end{pmatrix}$

$\Rightarrow p + 2q = u$ and $3p + 6q = v$

so that $v = 3u$; equation of line is $y = 3x$

(ii) $\begin{pmatrix}1 & 2\\3 & 6\end{pmatrix}\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}w\\3w\end{pmatrix} \Rightarrow p + 2q = w$

$\Rightarrow q = \frac{1}{2}(w - p)$

ie equation is $y = \frac{w}{2} - \frac{x}{2}$

(iii) $\begin{pmatrix}1 & 2\\3 & 6\end{pmatrix}\begin{pmatrix}p\\q\end{pmatrix} = \begin{pmatrix}p\\3p\end{pmatrix} \Rightarrow p + 2q = p$ (so that $q = 0$),

(and $3p + 6q = 3p$)

Thus all points on the x-axis map to points with the same x-coordinate.