

Matrices – Q39: Simultaneous Eq'ns [Practice/E] (4/6/21)

Use matrices to find the plane containing the points
 $(2, -1, 4)$, $(-3, 4, 2)$ and $(1, 0, 5)$

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Solution

Let the eq'n of the plane be $ax + by + cz = 1$

$$\text{Then } 2a - b + 4c = 1 \quad (1)$$

$$-3a + 4b + 2c = 1 \quad (2)$$

$$a + 5c = 1 \quad (3)$$

$$\text{Thus } \begin{pmatrix} 2 & -1 & 4 \\ -3 & 4 & 2 \\ 1 & 0 & 5 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{vmatrix} 2 & -1 & 4 \\ -3 & 4 & 2 \\ 1 & 0 & 5 \end{vmatrix} = 2(20) - (-3)(-5) + (-18) = 7$$

$$\begin{pmatrix} 2 & -1 & 4 \\ -3 & 4 & 2 \\ 1 & 0 & 5 \end{pmatrix}^{-1} = \frac{1}{7} \begin{pmatrix} 20 & 17 & -4 \\ 5 & 6 & -1 \\ -18 & -16 & 5 \end{pmatrix}^T$$

$$\text{So } \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 20 & 5 & -18 \\ 17 & 6 & -16 \\ -4 & -1 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

$$\text{and hence } a = \frac{1}{7}(20 + 5 - 18) = 1,$$

$$b = \frac{1}{7}(17 + 6 - 16) = 1, \quad c = \frac{1}{7}(-4 - 1 + 5) = 0$$

and so the eq'n of the plane is $x + y = 1$