Matrices – Q32: Shears [Problem/M] (3/6/21)

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Solution

The determinant will equal 1, in the case of a shear.

$|M^{-1}| = |M|$ and $\text{tr}(M^{-1}) = tr(M)$

[as $M^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$, if $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$] , so that M^{-1} will also represent a shear. It will be in the opposite direction to that represented by M.