

Matrices – Q31: Shears [Problem/M] (3/6/21)

Consider the matrix $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$, which represents a shear. Show that it is not possible for all of the elements of the matrix to be positive. [It can be assumed that $\text{tr}M = 2$.]

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Solution

$$ad - bc = 1 \text{ \& } a + d = 2$$

$$\Rightarrow a(2 - a) - bc = 1$$

$$\Rightarrow -bc = a^2 - 2a + 1 = (a - 1)^2$$

If b & c are both positive, then $(a - 1)^2 < 0$, which isn't possible.