Matrices – Q26: Determinants [Problem/H](2/6/21)

Given that a 3×3 determinant can always be reduced to triangular form (in the same way as simultaneous equations), to produce a multiple of $\begin{vmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{vmatrix}$, show that it can be further reduced to a multiple of the Identity matrix. [Obviously this is an academic exercise, as the determinant can be evaluated as soon as triangular form has been reached.]

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Solution

From $\begin{vmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{vmatrix}$, if $R1 \rightarrow R1 - a(R2)$, we get $\begin{vmatrix} 1 & 0 & b - ac \\ 0 & 1 & c \\ 0 & 0 & 1 \end{vmatrix}$

Then, $C3 \rightarrow C3 - c(C2)$ gives $\begin{vmatrix} 1 & 0 & b - ac \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

and finally $R1 \rightarrow R1 - (b - ac)(R3)$ gives $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

[Note that no further factors have had to be taken outside the determinant - as expected, since $\begin{vmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{vmatrix} = 1$]