Matrices – Q24: Determinants [Practice/M](2/6/21)

Write the determinant $\begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix}$ as a product of linear factors.

Write the determinant $\begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix}$ as a product of linear factors.

Solution

Replacing row 1 with row 1 - row 2,

$$D = \begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix} = \begin{vmatrix} 0 & x^2 - y^2 & x^4 - y^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix}$$

$$= (x^2 - y^2) \begin{vmatrix} 0 & 1 & x^2 + y^2 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix}$$

Similarly, replacing row 2 with row 2 - row 3,

$$D = (x^2 - y^2)(y^2 - z^2) \begin{vmatrix} 0 & 1 & x^2 + y^2 \\ 0 & 1 & y^2 + z^2 \\ 1 & z^2 & z^4 \end{vmatrix}$$

$$= (x^2 - y^2)(y^2 - z^2)(y^2 + z^2 - [x^2 + y^2])$$

$$= (x^2 - y^2)(y^2 - z^2)(z^2 - x^2)$$

$$= (x - y)(x + y)(y - z)(y + z)(z - x)(z + x)$$