

# Matrices – Q23: Determinants [Practice/M](2/6/21)

Factorise the determinant  $\begin{vmatrix} x^2 - x & y^2 - y & z^2 - z \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$

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### Solution

$$C2 \rightarrow C2 - C1 \text{ \& } C3 \rightarrow C3 - C1 \Rightarrow$$

$$\begin{aligned} & \begin{vmatrix} x^2 - x & y^2 - y - x^2 + x & z^2 - z - x^2 + x \\ x & y - x & z - x \\ 1 & 0 & 0 \end{vmatrix} \\ &= \begin{vmatrix} x^2 - x & (y^2 - x^2) - (y - x) & (z^2 - x^2) - (z - x) \\ x & y - x & z - x \\ 1 & 0 & 0 \end{vmatrix} \\ &= \begin{vmatrix} x^2 - x & (y - x)(y + x - 1) & (z - x)(z + x - 1) \\ x & y - x & z - x \\ 1 & 0 & 0 \end{vmatrix} \\ &= (y - x)(z - x) \begin{vmatrix} x^2 - x & y + x - 1 & z + x - 1 \\ x & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} \\ &= (y - x)(z - x)\{y + x - 1 - (z + x - 1)\} \\ &= (y - x)(z - x)(y - z) \end{aligned}$$

Alternatively:

$$\begin{aligned} R1 \rightarrow R1 + R2 & \Rightarrow \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} \\ C2 \rightarrow C2 - C1 \text{ \& } C3 \rightarrow C3 - C1 & \Rightarrow \begin{vmatrix} x^2 & y^2 - x^2 & z^2 - x^2 \\ x & y - x & z - x \\ 1 & 0 & 0 \end{vmatrix} \end{aligned}$$

$$\begin{aligned} &= (y - x)(z - x) \begin{vmatrix} x^2 & y + x & z + x \\ x & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} \\ &= (y - x)(z - x)(y - z) \end{aligned}$$