Matrices – Q22: Inverses [Problem/M](2/6/21)

Assuming that $(AB)^T = B^T A^T$, prove that $(A^T)^{-1} = (A^{-1})^T$

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Solution

Let $B = (A^T)^{-1}$, so that $BA^T = I$ (1)

Result to prove: $B = (A^{-1})^T$

[Noting that this is equivalent to $B^T = A^{-1}$, it seems promising to involve B^T]

From (1), $(BA^T)^T = I^T = I$, so that $AB^T = I$,

and hence $B^T = A^{-1}$ and $B = (A^{-1})^T$, as required.